Exam
Contact Mechanics and Elements of Tribology

Your name ____________________

February 28, 2020
(reply in English or in French)

1. You are polishing a (very hard) Nickel Base superalloy with hardness $H = 500$ MPa. The specimen has a section of $S = 1$ cm^2 and is placed on a rotary polishing machine at 10 cm from the center.The machine rotates at 300 rotations per minute (RPM) and you apply a load of $P = 100$ N. The abrasive wear law is $\Delta V/\Delta l = K'A$, where A is the real contact area, and wear coefficient is $K' = 10^{-4}$. How long will it take to polish $h = 0.1$ mm of the specimen?

2. Two cylinders are in contact along their circular sections (see Fig. 3). The height of cylinders are $H_1 = 2$ m and $H_2 = 1$ m, the radius is the same 30 cm, but cylinder have different elastic properties $E_1 = 1$ GPa, $\nu_1 = 0.2$ and $E_2 = 4$ GPa, $\nu_1 = 0.5$. The cylinders are squeezed together by an axial force $F = 1$ kN. (a) Find the contact pressure distribution $p(x, y)$. (b) Find the displacement u_z of the contact section $z = z_c$. (c) How the solution would change if the contacting sides of the cylinders are very slightly curved outwards?

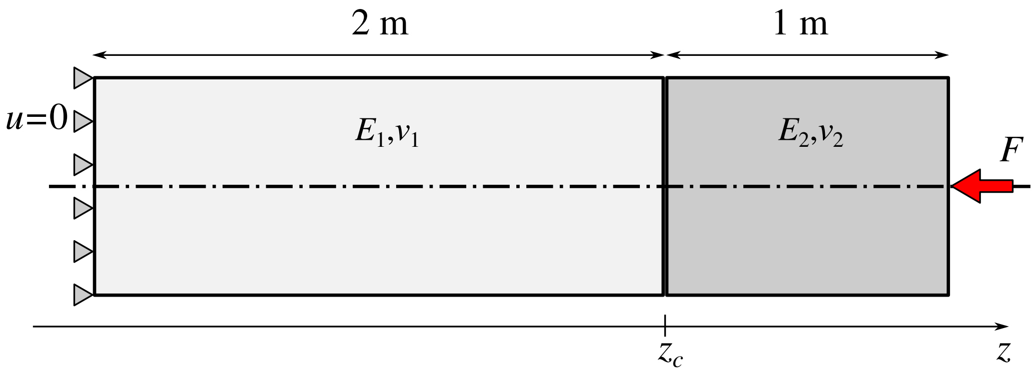

Figure 1: Two cylinders contact.

3. Two elastically different solids (plane strain) are brought in normal contact (only normal load is applied), the friction coefficient is non-zero $\mu \neq 0$. Using Flamant's solution, deduce the relation between elastic moduli E_1, ν_1 and E_2, ν_2 that would ensure the absence of relative tangential slip between solids in the contact interface, which would result in absence of tangential tractions in contact interface, i.e. $\sigma_t = 0$.

4. Two equal vertical compressive forces P (linear force density with units [N/m]) act on a half-plane at locations $x = 0$ and $x = L$. For a given isotropic elastic material determine what is the induced tangential (horizontal) displacement at location (a) $x = L/2$, (b) $x = L/4$, (c) $x = -L$ and (d) $x = 2L$. See Fig. 2.

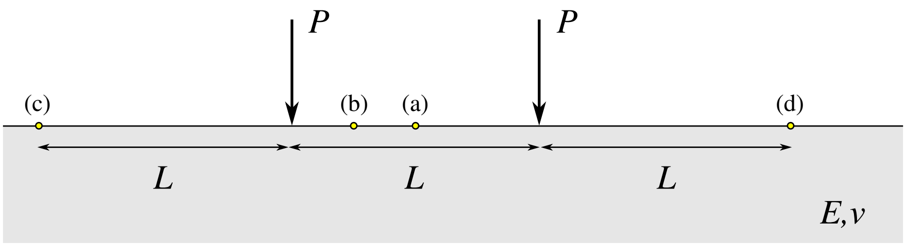

Figure 2: Two forces acting on the elastic half-plane.

5. (a) How would the expression for the stress components change in Flamant's solution if we derive them in plane stress (instead of considering the plane strain problem as was done in the class)? (b) Why this solution is less frequently used for contact problems?

6. In plane strain configuration: (a) Which profile the contacting solids should have in order to produce a uniform contact-pressure distribution in contact interface under the action of force N? (b) Is the solution unique? (c) Could you suggest a profile-geometry which results in a uniform pressure distribution for an arbitrary normal load?

7. A rolling bearing with spherical balls of radius $r = 4$ mm, the inner ring and the outer ring (tubes of circular section) of outer radius $R = 5.2$ cm and internal radius $R = 6$ cm, respectively, should hold a radial load of $F = 20$ kN; the balls and the rings are made of a steel $E = 2.1 \cdot 10^{11}$ Pa, Poisson's ratio $\nu = 0.3$. Estimate the maximal contact pressure in the system.

8. Two rigid needles are located on a flat rigid foundation and are oriented with tips pointing in OZ direction. These needles come in contact with an elastic half-space $E = 10$ GPa, $\nu = 0.5$ whose outward normal is given by $-\underline{e}_z$, the coordinates of tips $\{x, y, z\}$ for needles named by $\{A, B\}$ are give by $\{0, 0, 1\}, \{1, 0, 0.9\}$ cm, respectively. The higher needle is in contact with the elastic half-space. Compute the normal load needed to bring the lower needle in contact too.

9. Explain Amontons-Coulomb's friction law from micromechanical considerations (roughness and the difference between the real and the apparent contact area).

10. A block of mass m slides on a flat horizontal surface. The initial velocity of the point is V_0. The static coefficient of friction is μ_s, the kinetic coefficient of friction is μ_k. Find the distance at which the point will stop.

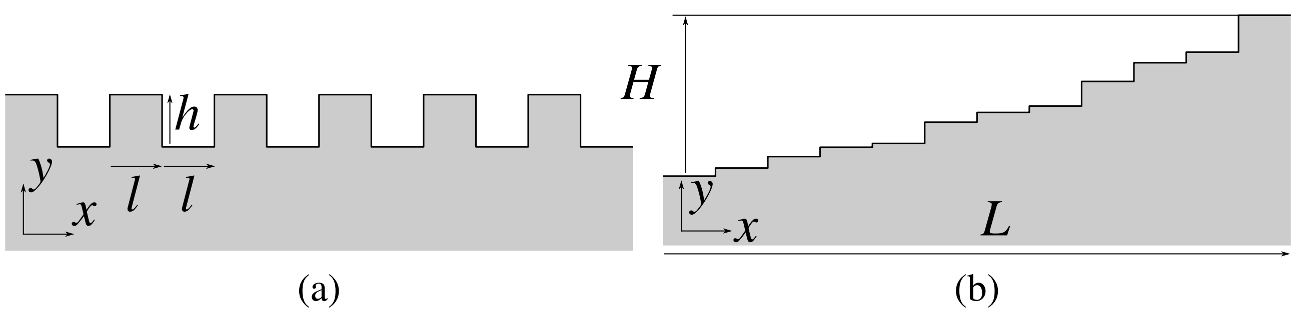

Figure 3: Step geometry (a) periodic step structure, (b) monotonous step surface defined over a finite length.

11. Compute the probability density function and the root mean absolute slope of an infinite regular profile shown in Fig. 3(a).

12. Compute the mean absolute slope of a monotonic and finite-size profile shown in Fig. 3(b).

13. What is the curvature of planar asperities on a surface $y(x) = A\sin(kx + \phi_0)$?

14. For a one-dimensional system of two springs shown in Fig. 4 formulate the penalized energy functional and solve the problem for the following boundary conditions: displacement on the left $u_1 = 0$ and $u_4 = -1.5g_0$. Demonstrate that for the penalty parameter $\epsilon \to \infty$ the solution tends to the exact solution, which should be also found.

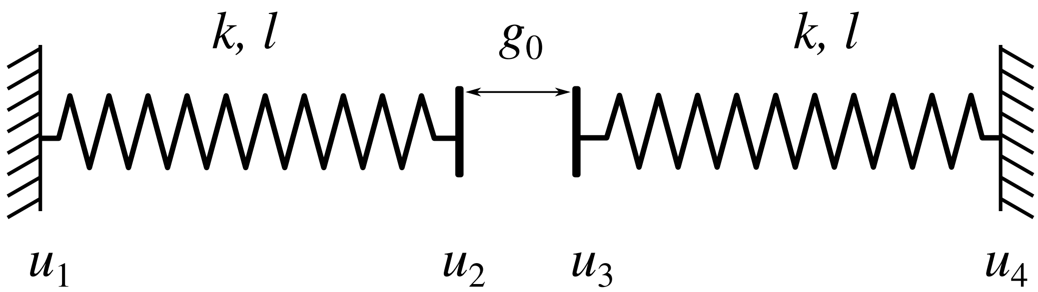

Figure 4: Contact system of two springs of equal length and stiffness

15. Consider a parallel-step slider bearing, see Fig.5. For the given bearing configuration, i.e. known geometrical parameters l, n, h_0, s and the velocity u:

    (a) Calculate the **normal force** acting on the bottom plane ($z = 0$). *Hint: use the formula derived in the lectures for pressure distribution along the x-coordinate.*

    (b) Find the **shear stress** on the bottom plane ($z = 0$). *Hint: use the formula for shear stress in a Newtonian fluid and recall the velocity profile for the combined Couette and Poisseuille flow.*

    (c) Calculate the total **shear force** acting on the bottom plane ($z = 0$).

    (d) Calculate the **friction coefficient** for the considered bearing.

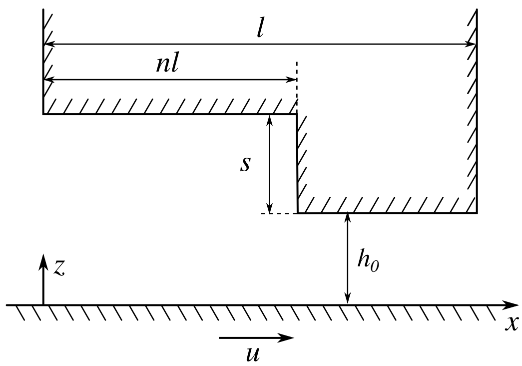

Figure 5: A sketch of the parallel-step slider bearing: l is the length of the bearing, nl is the length of the step, h_0 is the minimal film thickness and s is the height of the step. The upper part of the bearing is at rest, while the bottom plane is moving to the right with a constant velocity u.