Contact mechanics and elements of tribology

Lecture 4.
Micromechanical contact: roughness

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@ Centre des Matériaux
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Outline

- Introduction
- Measurement techniques
- Classifications
- Main characteristics
- PDF and PSD
- Random process model of roughness
- Computational roughness models
- Reading

Roughness

- **Natural and industrial surfaces are *rough*:**
  - processing
  - polishing
  - coating
  - microstructure
  - surface energy
  - deformation
  - aging
  - environment

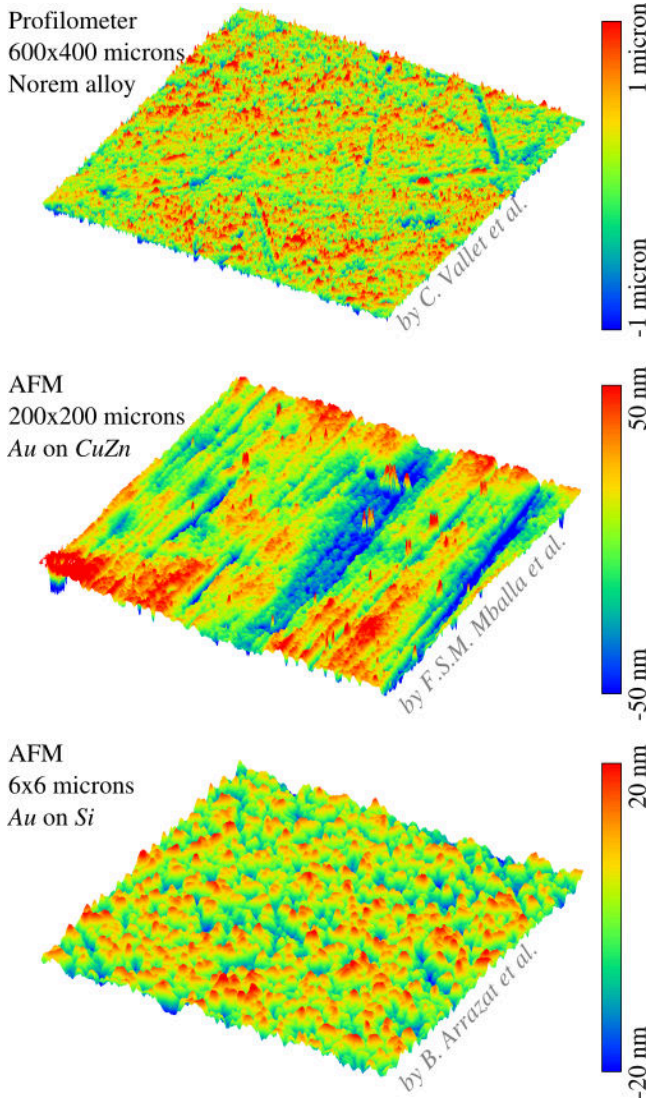

Fig. Examples of rough surfaces

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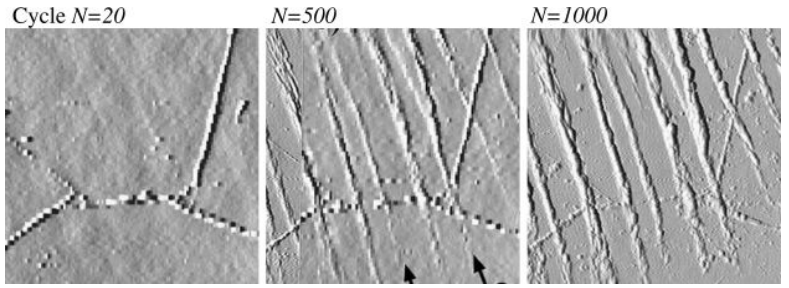

Fig. Persistent slip marks [1]

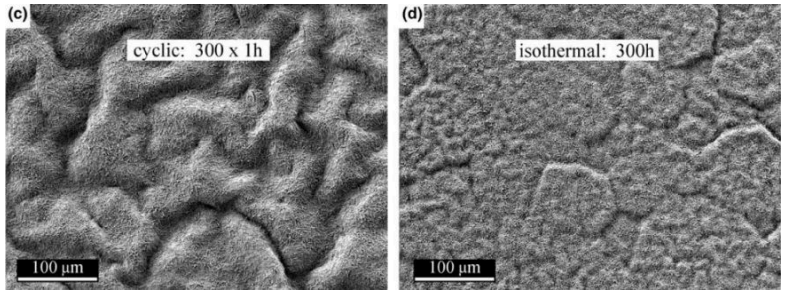

Fig. Rumpling (thermal cycling induced roughness in air)[2]

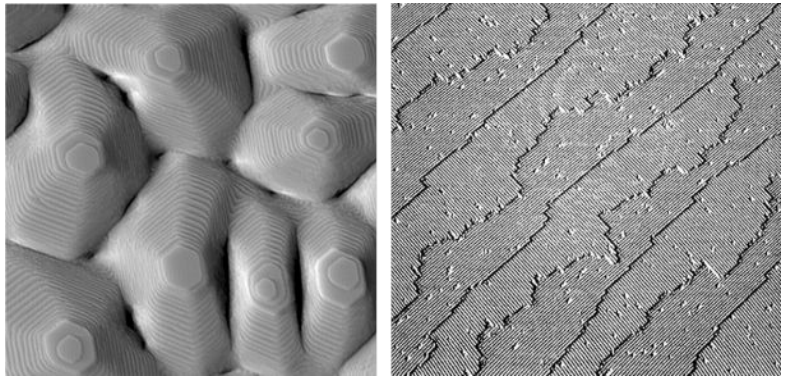

Fig. Epitaxial surface growth [3,4]

[1] J.Polák, J. Man & K. Orbtlík, Int J Fatigue 25 (2003)
[2] V.K. Tolpygo, D.R. Clarke, Acta Mat 52 (2004)
[3] M. Einax, W. Dieterich, P. Maass, Rev Mod Phys 85 (2013)

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- **Roughness affects:**
  - stress-strain state
  - dry friction
  - wear
  - adhesion
  - fluid flow
  - sealing
  - energy transfer

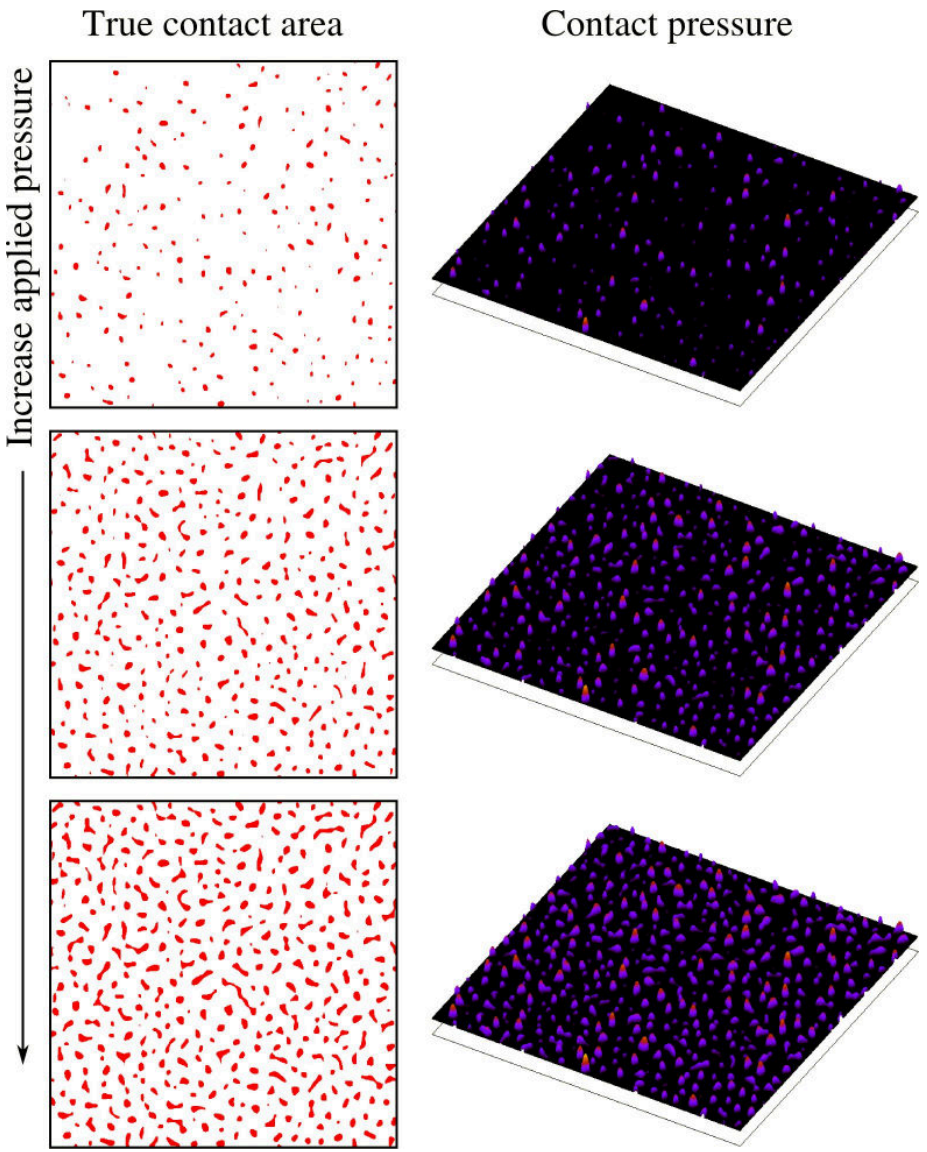

Fig. True contact area and stress fluctuations

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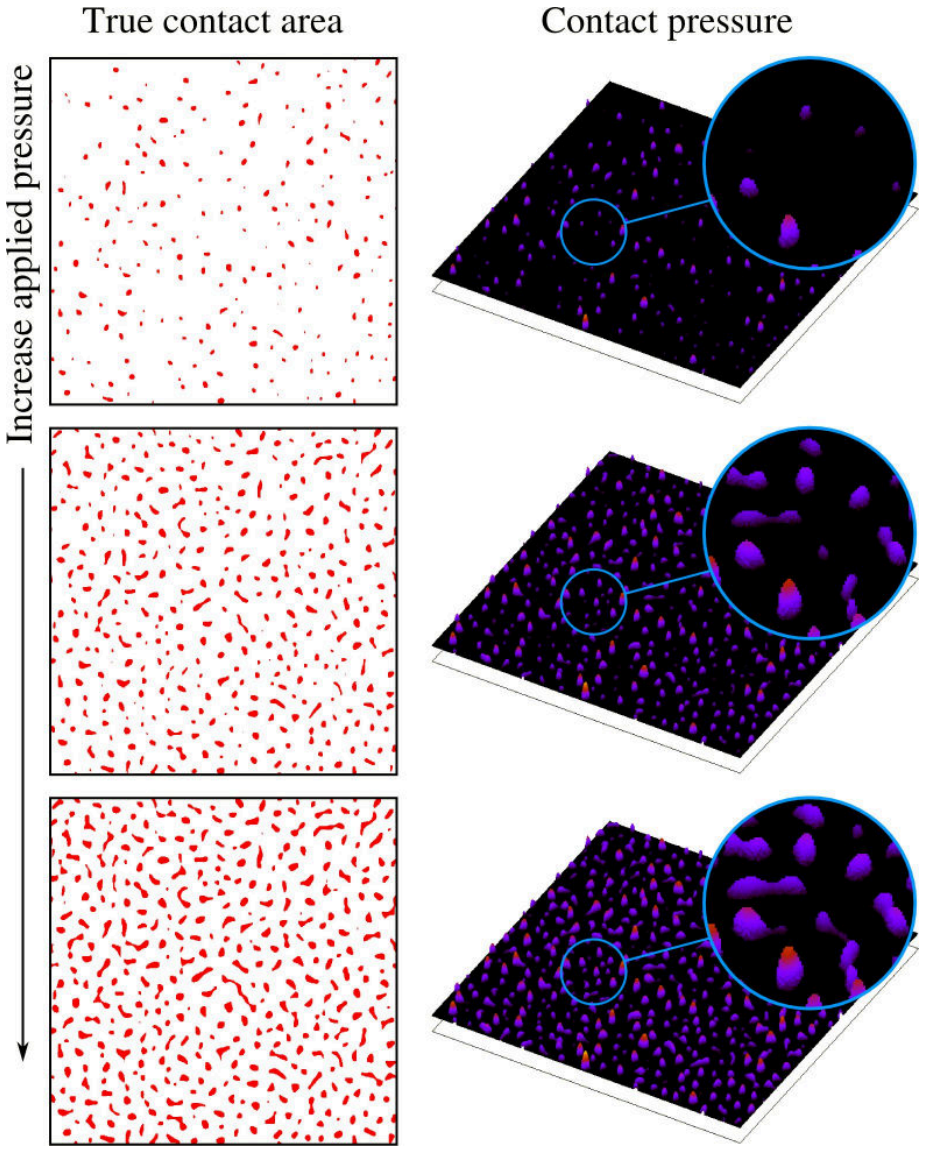

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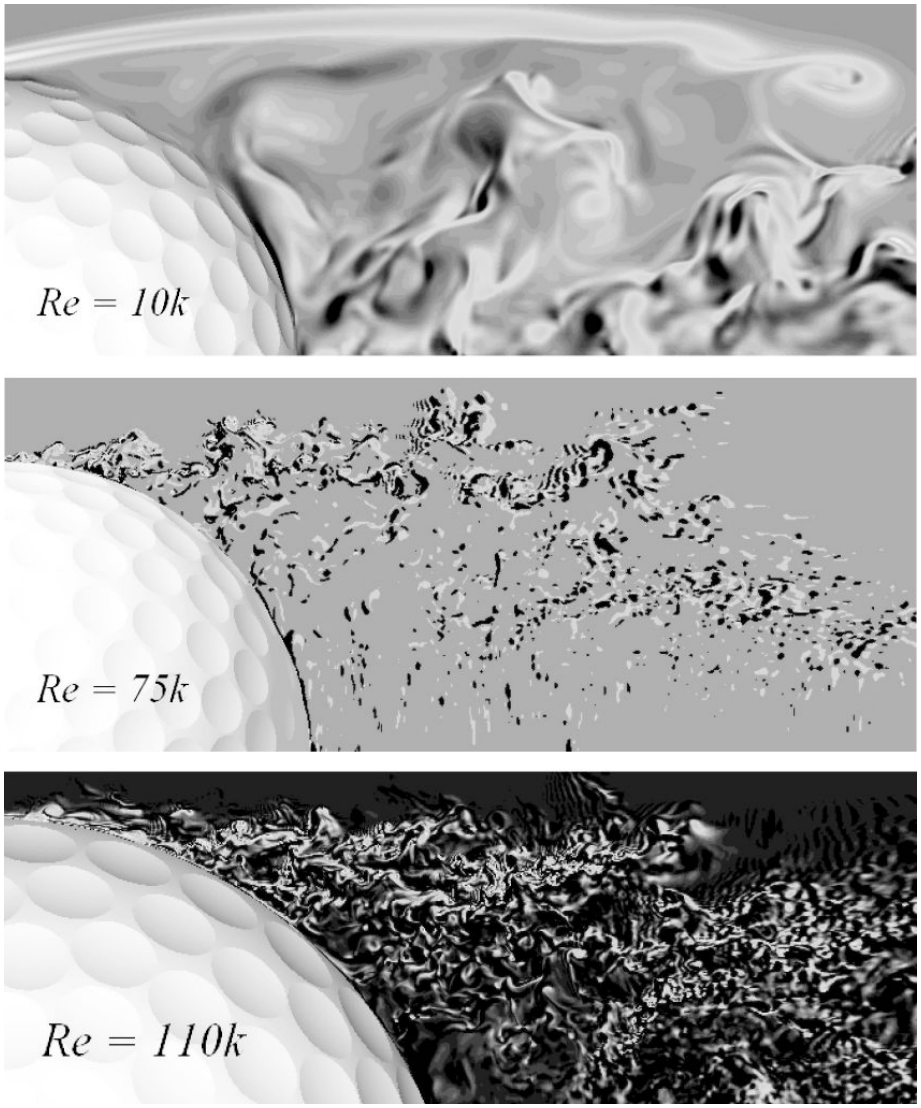

Fig. Numerical simulation of airflow around a (dimpled) golf ball [5]

[5] C.E. Smith, PhD thesis (2011)

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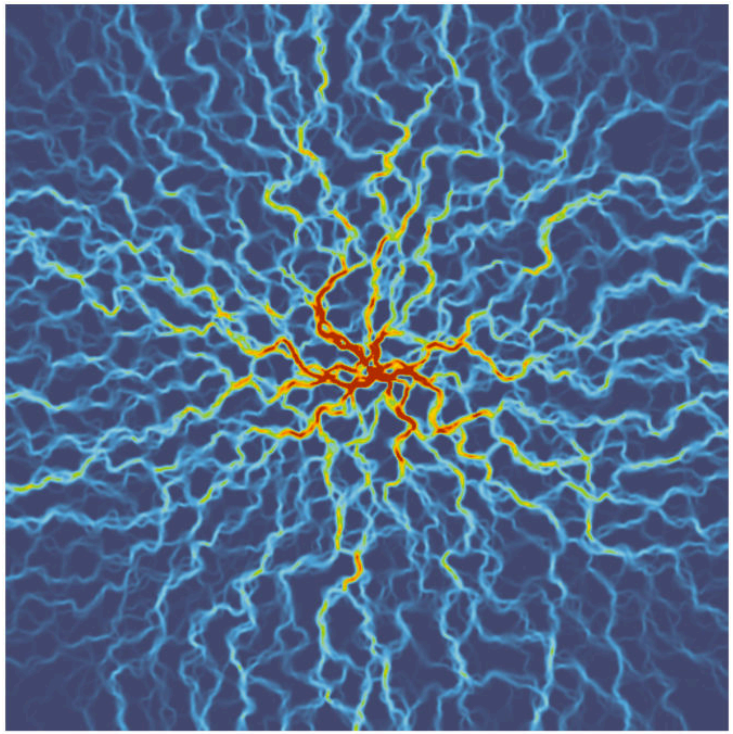


Fig. Fluid passage through free volume between rough surfaces

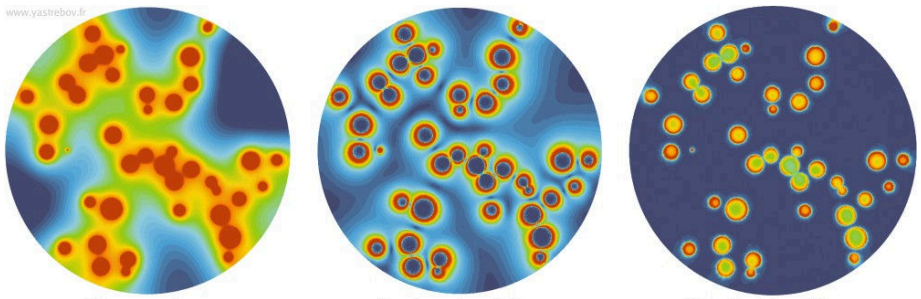
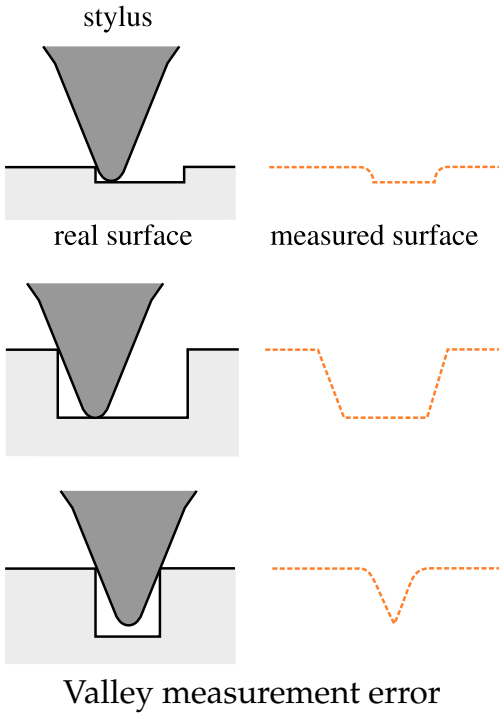


Fig. Heat transfer between rough surfaces (asperity-based model)

Surface metrology techniques

- Stylus measurements
  - Mechanical contact of a tip with surface
  - Force $\geq 3\mu g$, tip radius ≥ 50 nm
  - Mainly for profile measurements $y(x)$
- Optical measurements
  - Confocal (laser scanning) microscopy
    - highest lateral resolution
  - Interferometry (WLI):
    - highest vertical resolution
    - 10 to 100 times faster than CM
  - Scanning Electronic Microscopy (SEM):
    - in secondary electron emission
    - electrons penetrate in the matter → roughness smoothing
    - conducting materials
- Nano-contact measurements
  - Atomic Force Microscopy (AFM)
    roughness + adhesive and elastic properties
  - Scanning Tunneling Microscope (STM)

Stylus profilometer



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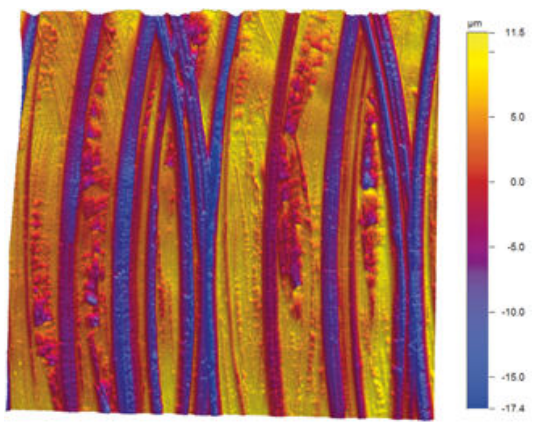
Stylus profilometer

Modern stylus profilometer
www.bruker.com



Roughness measurements ($\Delta z \approx 30$ μm)
www.icryst.com

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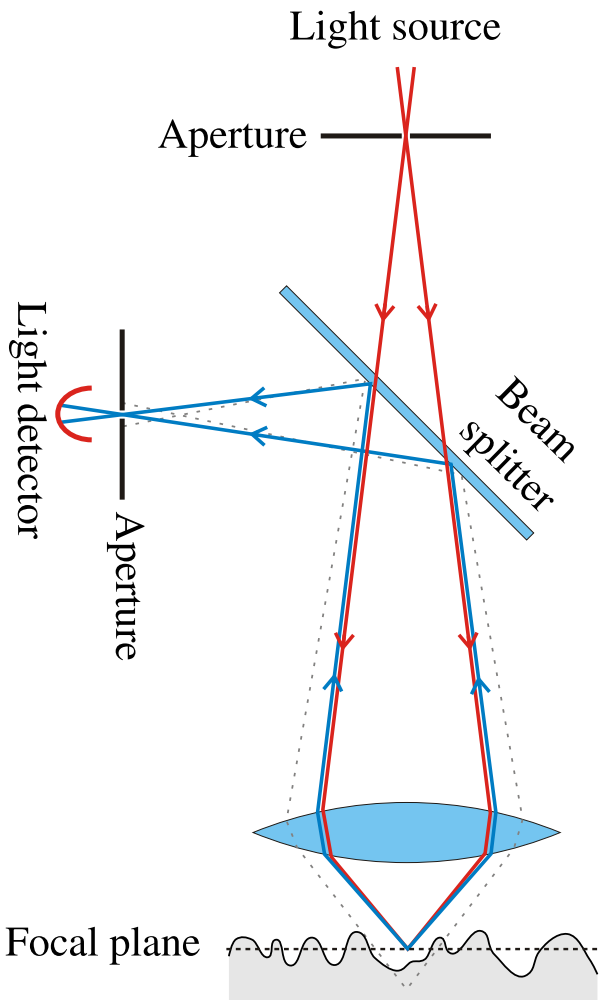
Confocal microscopy



Principle of confocal microscopy
adapted from www.wikipedia.org

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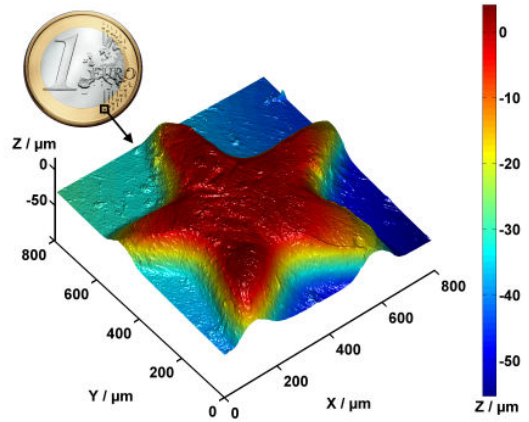
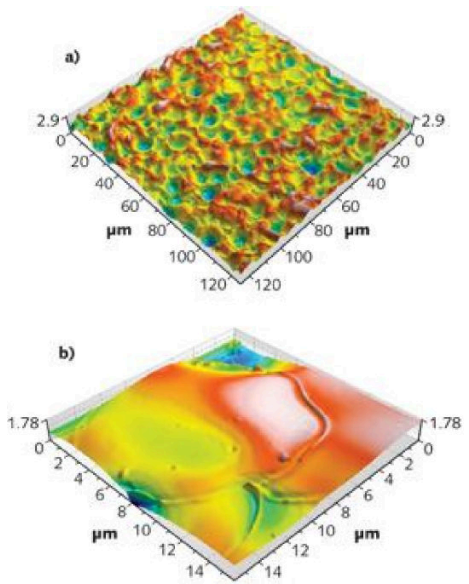


1 euro surface www.wikipedia.org



Stainless steel machined with micro-electric discharge www.laserfocusworld.org

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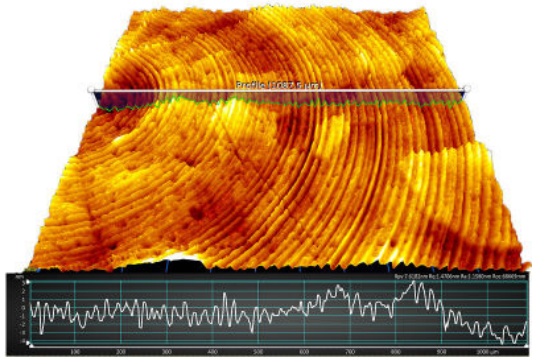
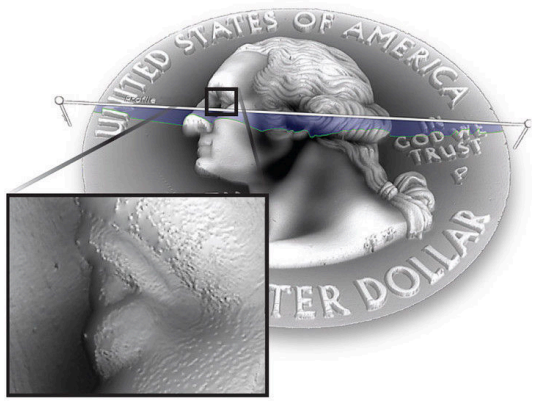
White Light Interferometry



Diamond-turned optics www.zygo.com



US quarter surface www.zygo.com

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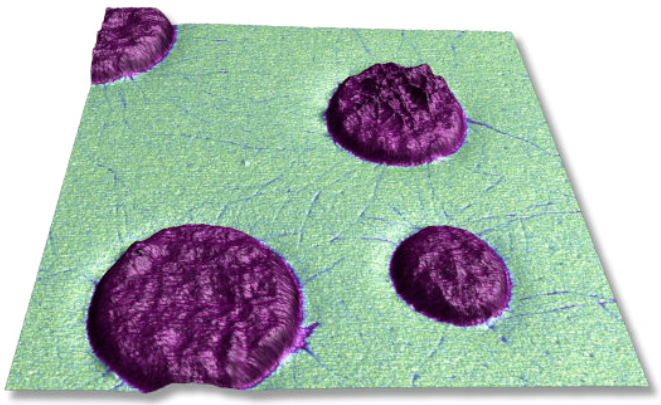
AFM

Modern AFM
www.bruker.com



Roughness and elastic moduli (color) of polymer blend
www.bruker.com

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STM

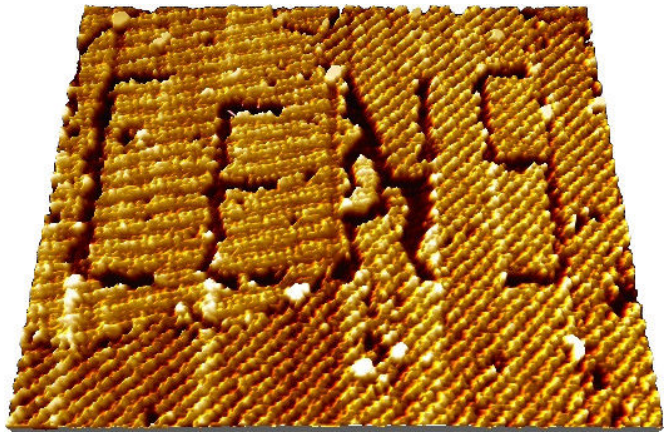
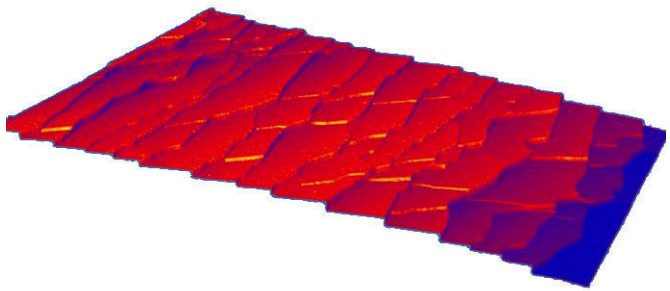


Fig. Center for NanoScience logo imprinted at atomic scale

www.cens.de



Atomic steps on platinum surface (500×500 nm)

www.icryst.com

Roughness: classification

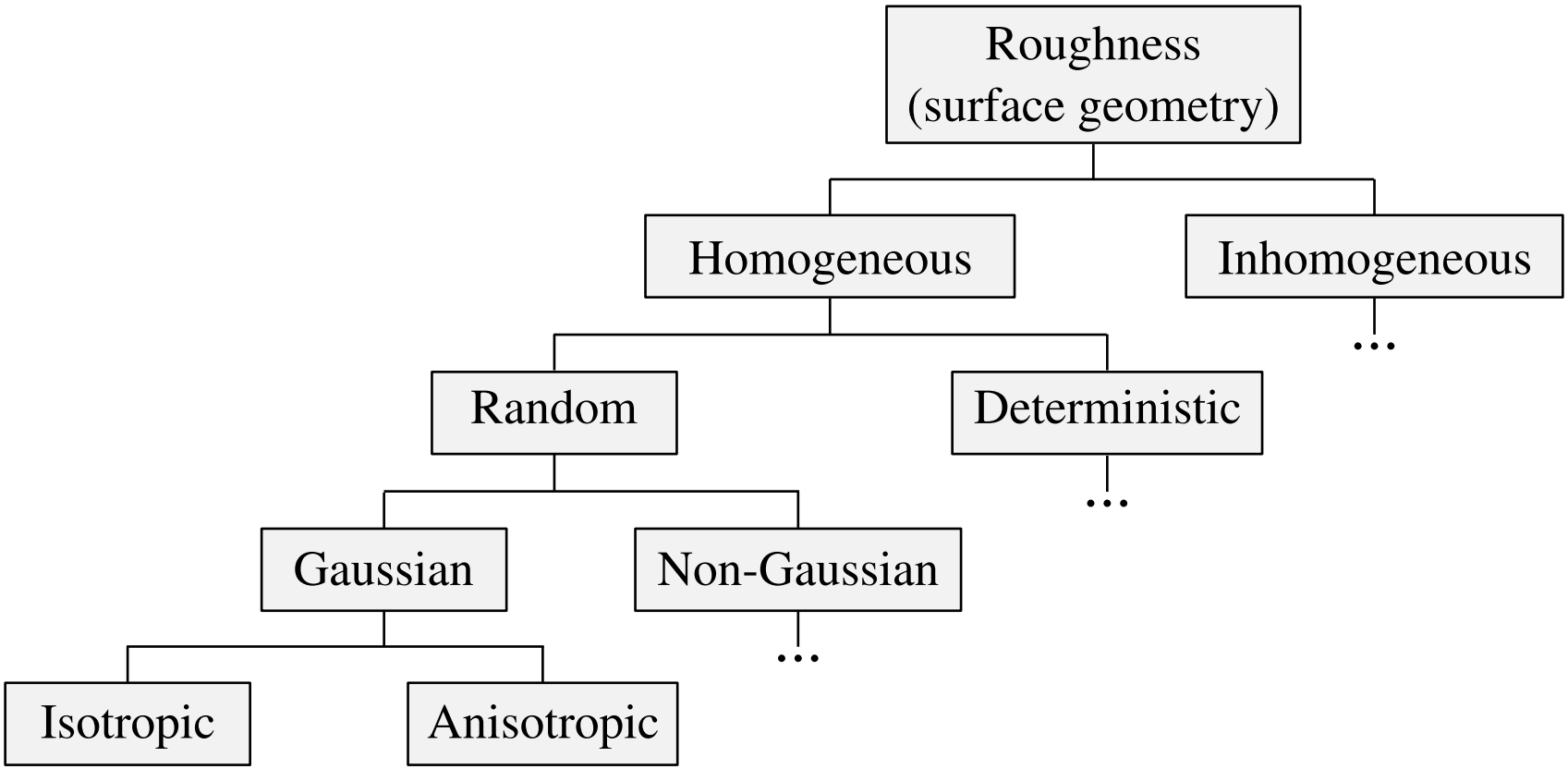


Fig. Roughness classification according to Nayak[1]

[1] Nayak, J. Lub. Tech. (ASME) 93:398 (1971)

Roughness: classification

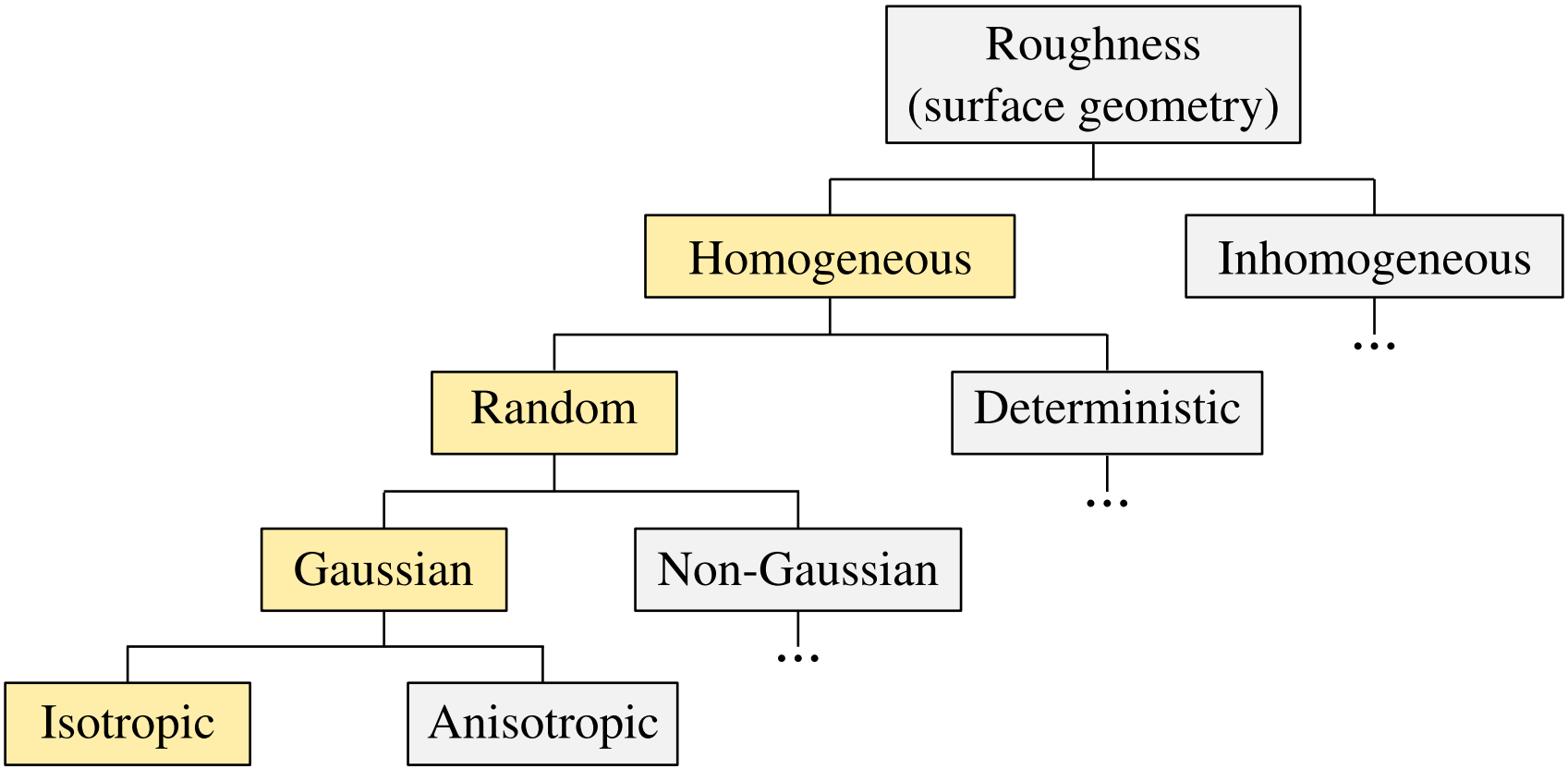


Fig. Roughness classification according to Nayak[1]

[1] Nayak, J. Lub. Tech. (ASME) 93:398 (1971)

Roughness and geometry/form

- Roughness vs geometry of surfaces
- Sometimes macroscopic geometry is substracted (filtered out): form→error of form→wavyness→roughness
- Non-trivial to remove macroscopic shape
- Most roughness measurement tools enable shape removal

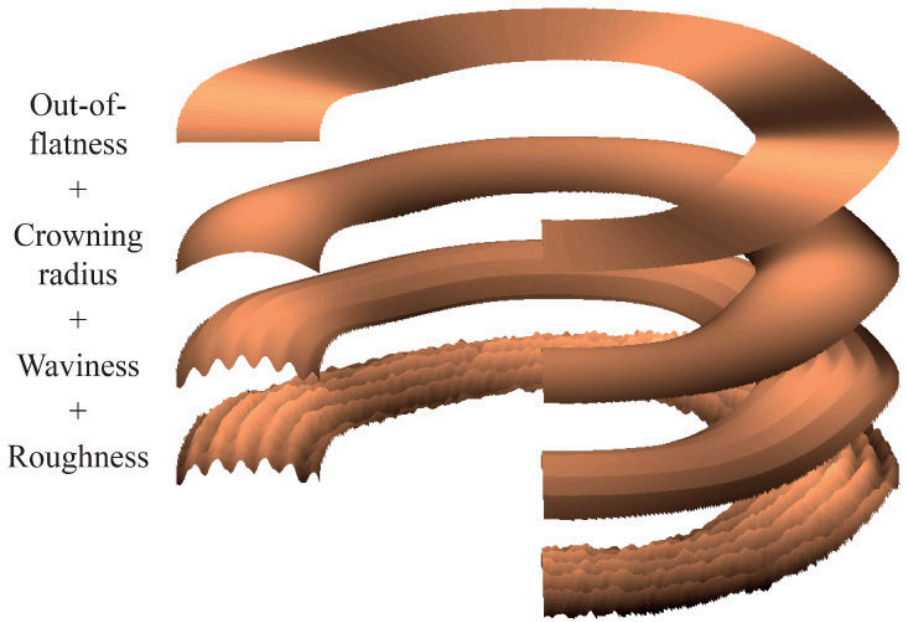


Fig. Circular metalic seal with turned copper surface[1]

[1] F.P. Rafòls, Licentiate Thesis, LTU 2016.

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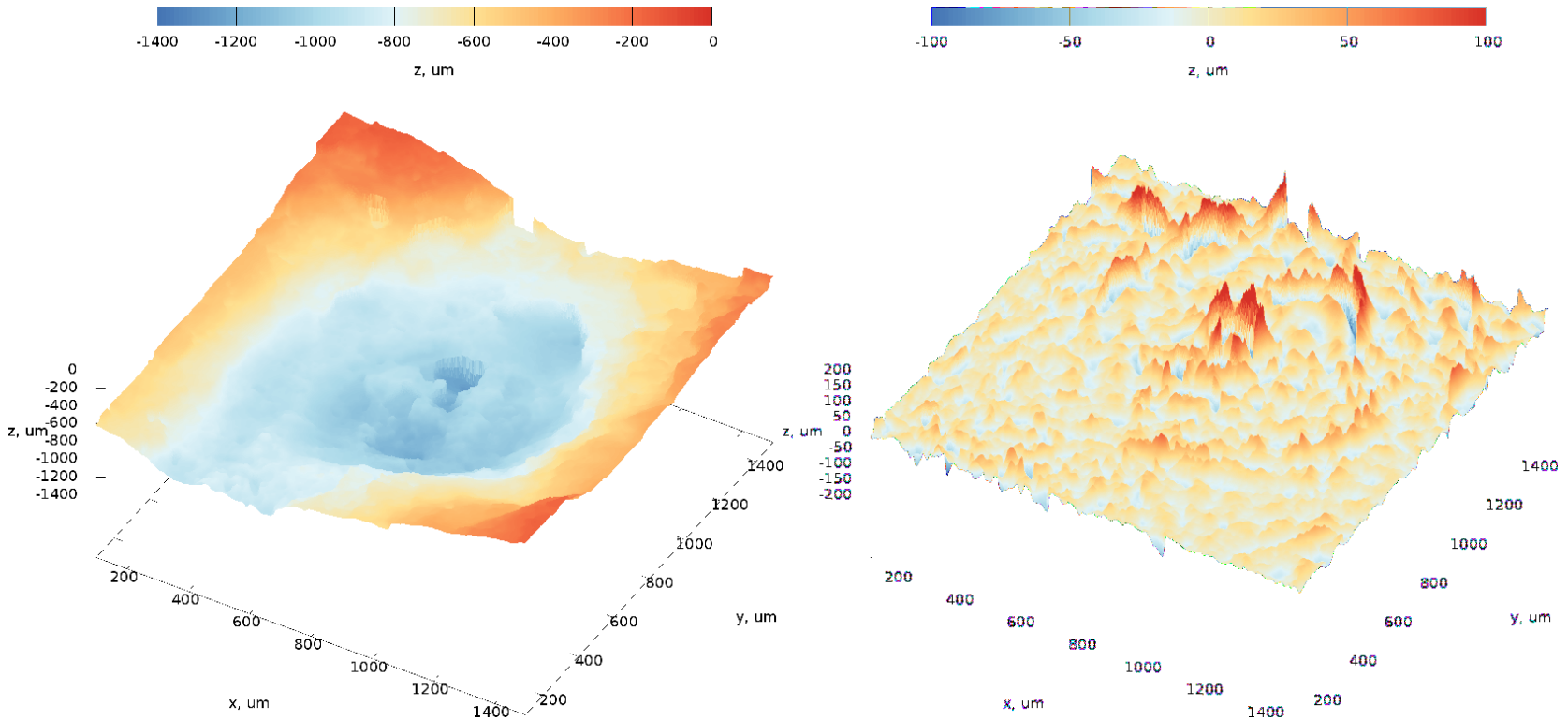

Fig. (left) impact crater, (right) shape is filtered out

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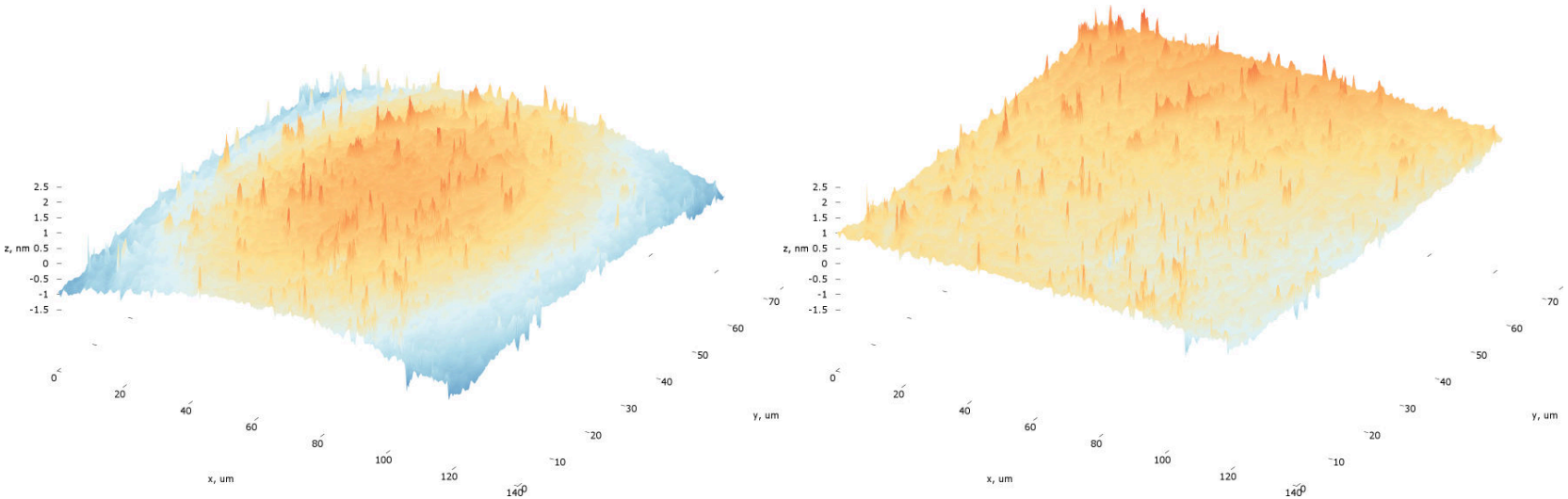


Fig. (left) spherical indenter, (right) $z = a(x^2 + y^2)$ shape is substracted

Integral quantities

- Average of absolute values [l.u.] (profile - R_a, surface - S_a)

$$S_a = \frac{1}{A}\int_A |z(x,y) - \bar{z}|\, dA, \quad S_a = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} |z_{ij} - \bar{z}|$$

- Standard deviation of height [l.u.] (σ or R_q for profile, S_q for surface)

$$\sigma = \sqrt{\frac{1}{A}\int_A (z(x,y) - \bar{z})^2\, dA}, \quad \sigma = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}(z_{ij} - \bar{z})^2}$$

- Maximal valley depth R_v,S_v, maximal peak height R_p,S_p [l.u.]
  very sensitive to sample area
- Skewness [adim] (γ_1 or R_{sk},S_{sk})

$$\gamma_1 = \frac{1}{A\sigma^3}\int_A (z(x,y) - \bar{z})^3\, dA, \quad \gamma_1 = \frac{1}{N^2\sigma^3}\sum_{i=1}^{N}\sum_{j=1}^{N}(z_{ij} - \bar{z})^3$$

- Kurtosis [adim] (κ or R_{ku},S_{ku})

$$\kappa = \frac{1}{A\sigma^4}\int_A (z(x,y) - \bar{z})^4\, dA, \quad \kappa = \frac{1}{N^2\sigma^4}\sum_{i=1}^{N}\sum_{j=1}^{N}(z_{ij} - \bar{z})^4$$

Integral quantities II

- Average of absolute value of gradient (slope) [adim] (profile - R_{dq}, surface - S_{dq})

$$S_{da} = \langle \left| \nabla z(x,y) - \overline{\nabla z} \right| \rangle = \frac{1}{A} \int_A \left| \nabla z(x,y) - \overline{\nabla z} \right| \, dA$$

$$S_{da} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \frac{z_{i+1,j} - z_{i,j} - \overline{\Delta z_x}}{\Delta x} \right| + \left| \frac{z_{i,j+1} - z_{i,j} - \overline{\Delta z_y}}{\Delta y} \right|$$

- Standard deviation of gradient (slope) [adim] (profile - R_{dq}, surface - S_{dq})

$$S_{dq} = \sqrt{\langle \left| \nabla z(x,y) - \overline{\nabla z} \right|^2 \rangle} = \sqrt{\frac{1}{A} \int_A \left| \nabla z(x,y) - \overline{\nabla z} \right|^2 \, dA}$$

$$S_{dq} = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \frac{z_{i+1,j} - z_{i,j} - \overline{\Delta z_x}}{\Delta x} \right| + \left| \frac{z_{i,j+1} - z_{i,j} - \overline{\Delta z_y}}{\Delta y} \right|^2}$$

- Often in integrated slope measurements a smoothing filter is used, for example, according to ASME B46.1 standard

$$\frac{\partial z}{\partial x} \approx \frac{1}{60\Delta x} (z_{i+3,j} - 9z_{i+2,j} + 45z_{i+1,j} - 45z_{i-1,j} + 9z_{i-2,j} - z_{i-3,j})$$

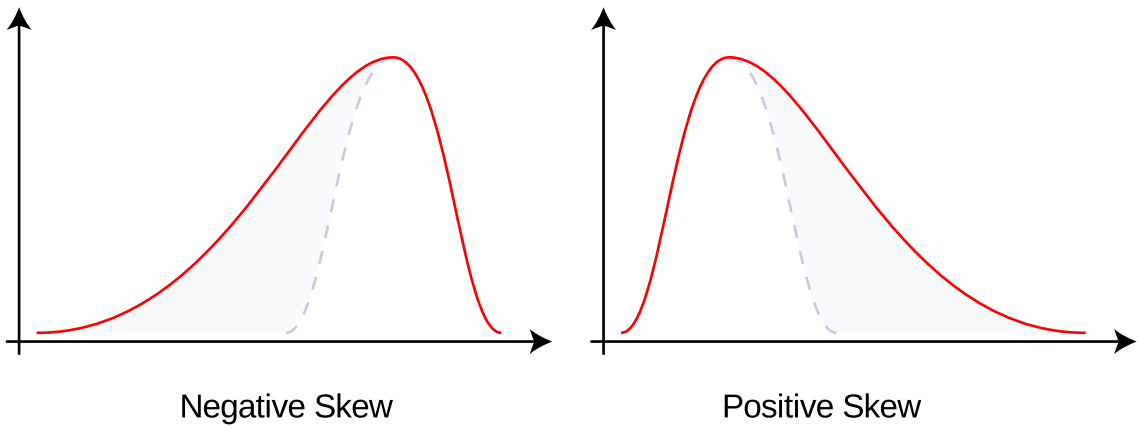
Main characteristics: probability density

- Probability density of heights $P(z)$
- Properties and moments

$$1 = \int_{-\infty}^{\infty} P(z)\,dz, \qquad \bar{z} = \int_{-\infty}^{\infty} zP(z)\,dz, \qquad \sigma = \sqrt{\int_{-\infty}^{\infty} (z-\bar{z})^2 P(z)\,dz}$$

$$\mu_q = \int_{-\infty}^{\infty} z^q P(z)\,dz \quad \text{then} \quad \mu_0 = 1,\ \mu_1 = \bar{z},\ \mu_2 = \sigma^2 + \bar{z}^2$$

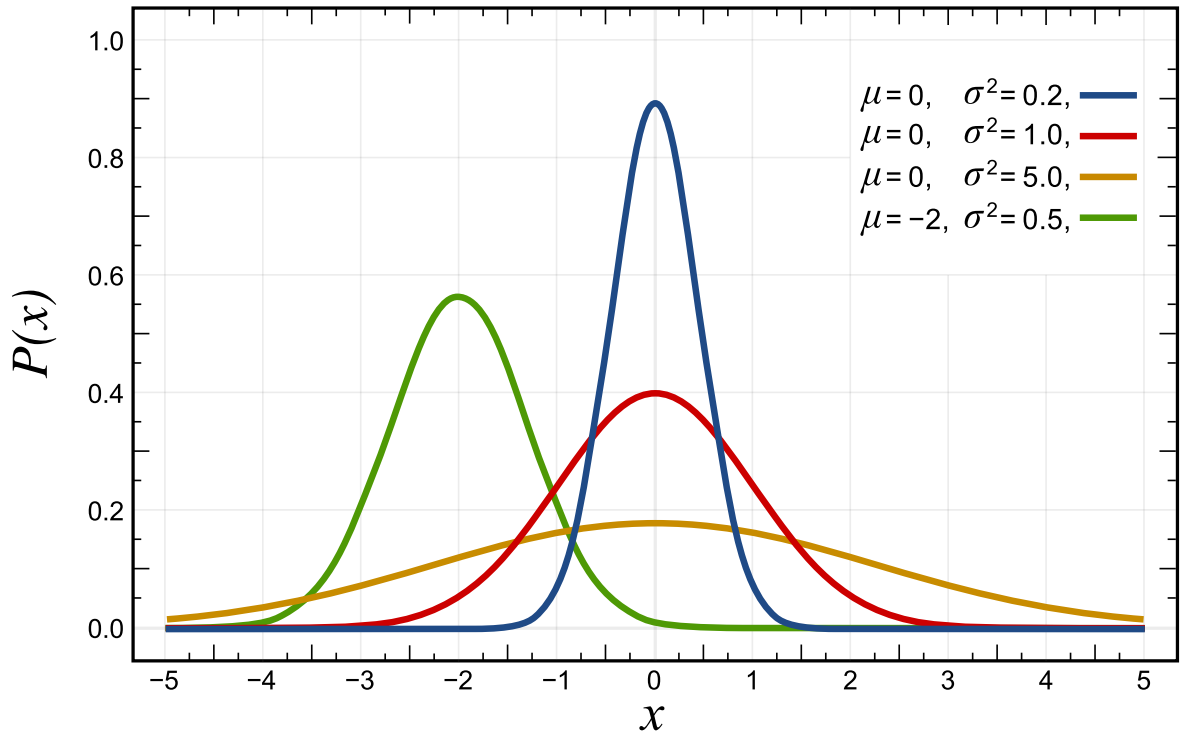
- Link to skewness



Main characteristics: probability density II

Distribution examples

- Normal (Gaussian): $P(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left[-\dfrac{(x-\mu_1)^2}{2\sigma^2}\right], x \in \mathbb{R}$

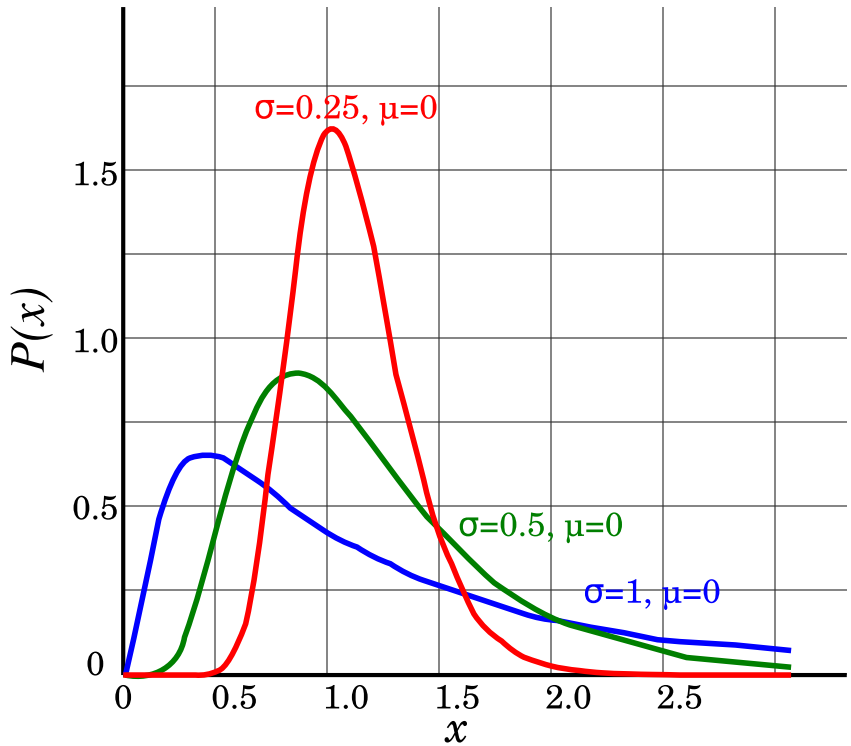


adapted from wikipedia.org

Main characteristics: probability density II

Distribution examples

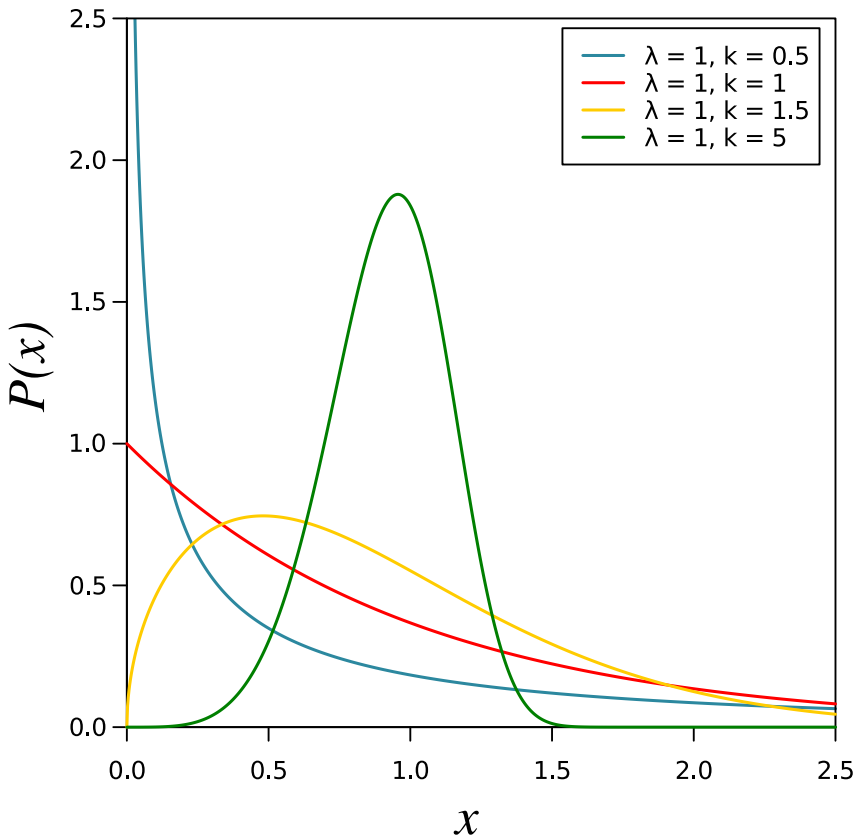
- Lognormal: $P(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\log(x)-\mu_1)^2}{2\sigma^2}\right] \; x \in \mathbb{R}^+$



adapted from wikipedia.org

Distribution examples

- Weibull: $P(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp -(x/\lambda)^k \; x \in \mathbb{R}^+$

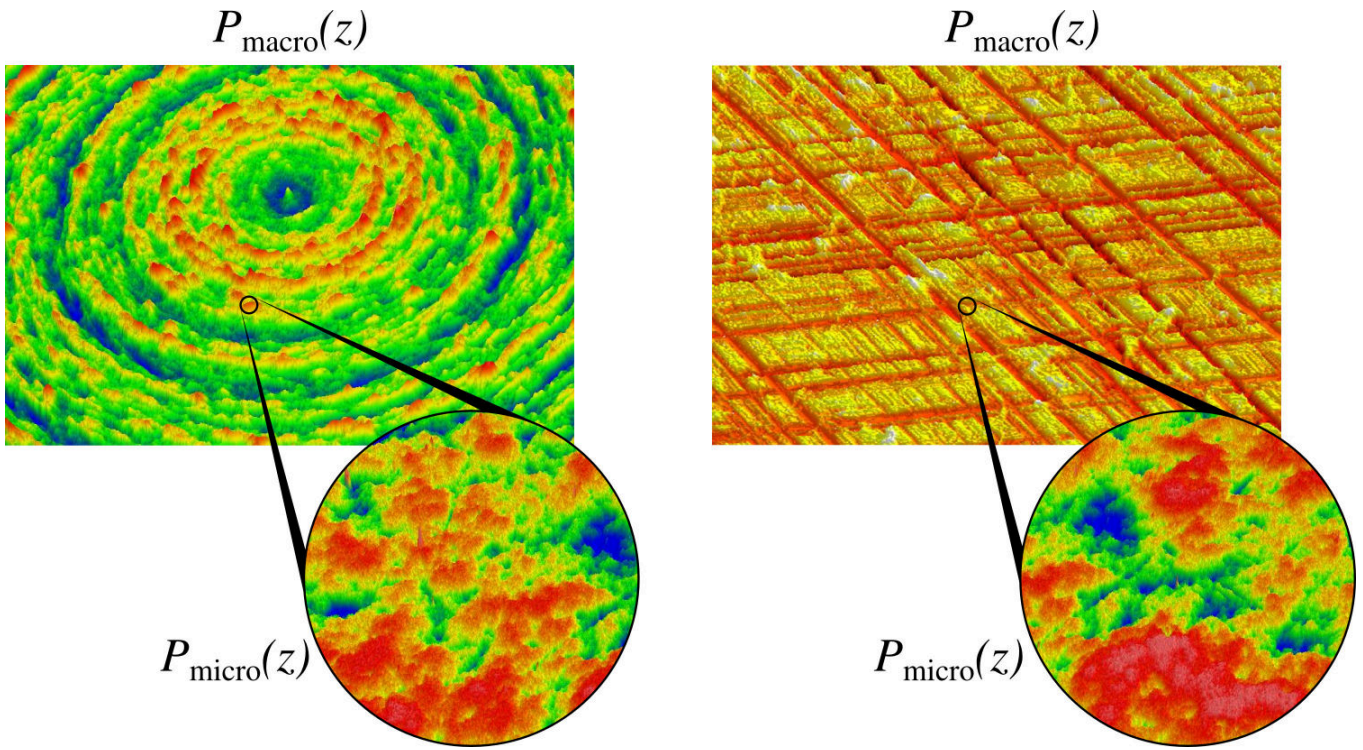


adapted from wikipedia.org

Main characteristics: comments on PDF

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \to 0$: negative curtosis



Macro- and microscopic roughness (left: diamond-turned surface, right: cross-hatched surface)
Images from www.zygo.com used

Main characteristics: comments on PDF

Real rough surfaces

- Turning, scratching, shaping changes macroscopic distribution P_{macro} but might keep microscopic distribution intact P_{micro}
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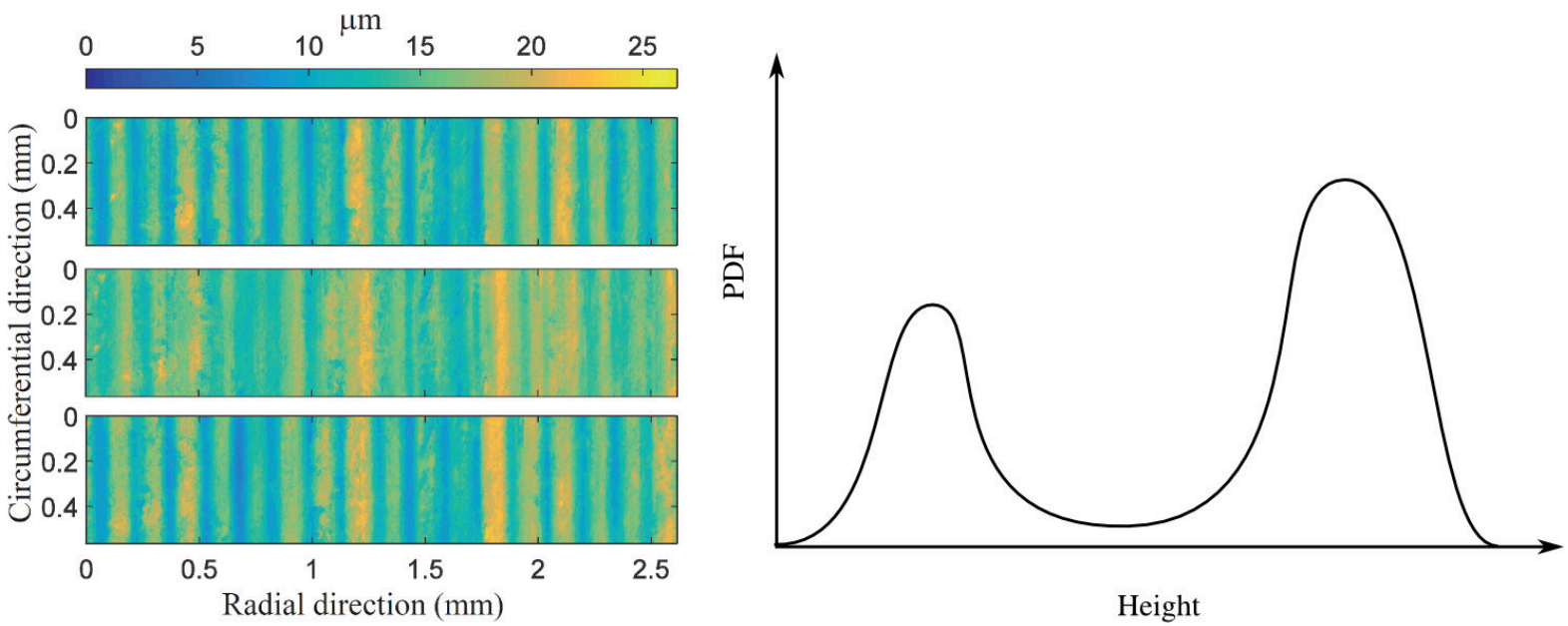

Turned surface topography and a sketch of height PDF
Pérez-Ràfols, Larsson, Almqvist, Tribol Int 94 (2016)

Main characteristics: comments on PDF

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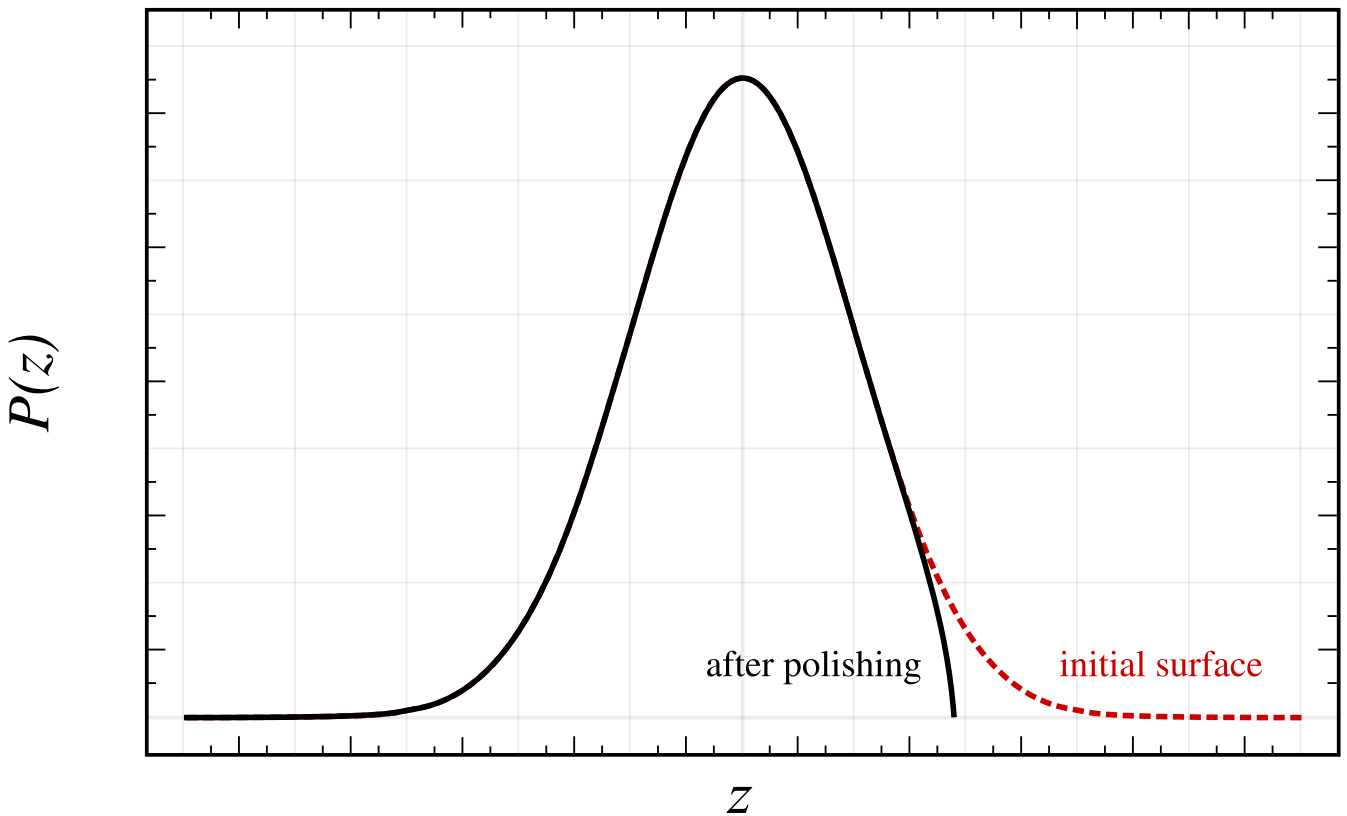

Initial surface with Gaussian PDF and surface after polishing

Main characteristics: comments on PDF

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Very fresh asphalt concrete

Normal asphalt concrete

Main characteristics: comments on PDF

Real rough surfaces

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Very fresh asphalt concrete

Normal asphalt concrete . . . with a bolt ☺

Main characteristics: comments on PDF

Real rough surfaces

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- Wear, polishing, flattening results in removal of the right distribution tail $P(z > z_0) \to 0$: negative curtosis

Old asphalt concrete with worn out bitumen

Main characteristics: comments on PDF

Real rough surfaces

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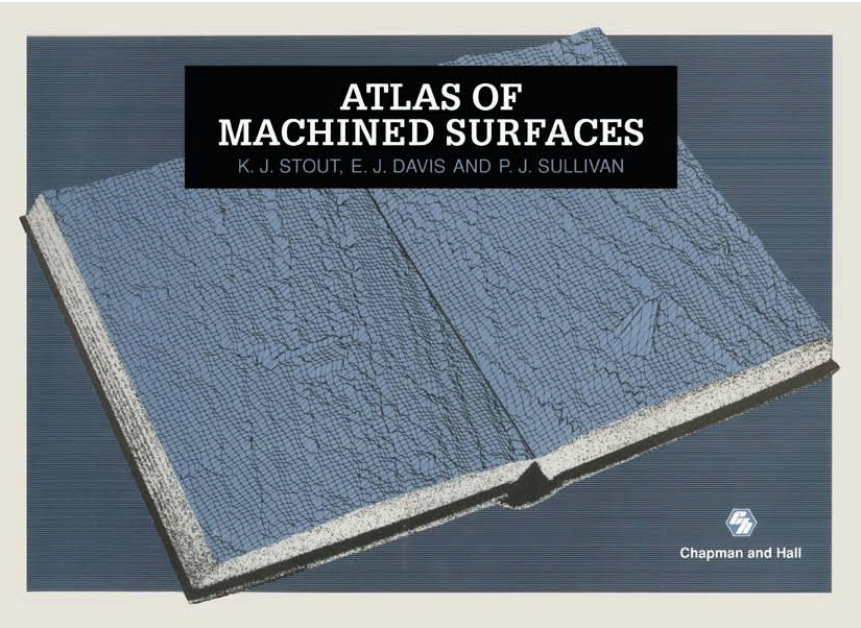


Atlas of machined surfaces (with height distributions)

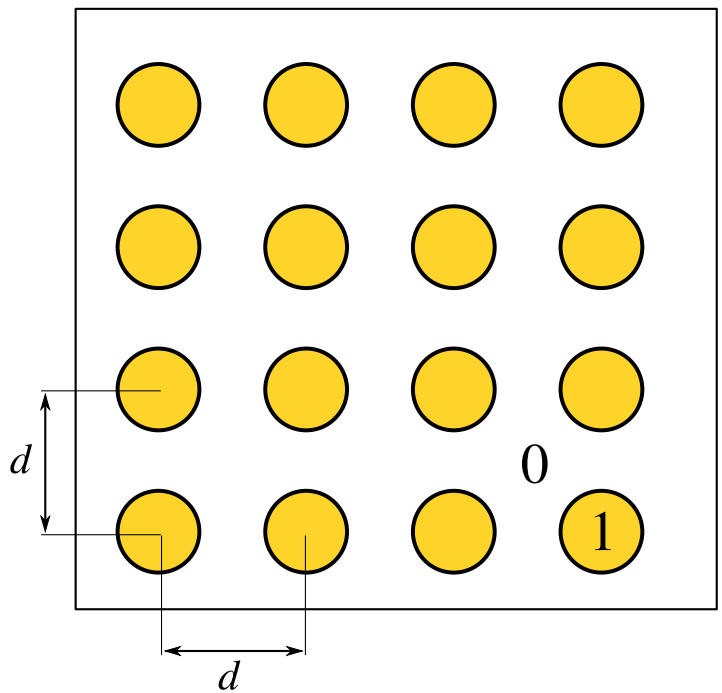
Autocorrelation function

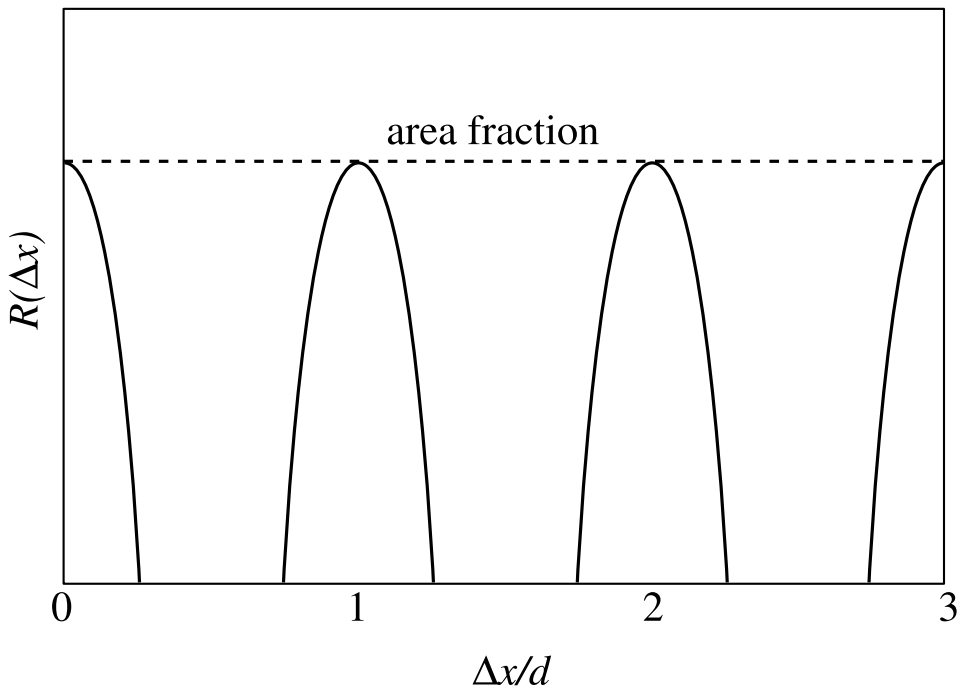
- Continuous autocorrelation function

$$R(\Delta x, \Delta y) = \lim_{L\to\infty} \frac{1}{L^2} \int_0^L \int_0^L z(x+\Delta x, y+\Delta y) z(x,y)\, dxdy$$

- Discrete autocorrelation function for a surface $N \times N$

$$R(\Delta x, \Delta y) = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} z(x+\Delta x, y+\Delta y) z(x,y)$$





Power spectral density (PSD)

- Recall: Fourier Transform: $\hat{f}(k) = \int_{-\infty}^{\infty} f(x) \exp(-2\pi i k x)\, dx$
- Recall: Discrete Fourier Transform: $\hat{f}_k = \sum_{n=0}^{N-1} x_n \exp(-2\pi i k n / N)$
- where x is the spatial coordinate, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength.
- PSD is the Fourier Transform of R

  $\Phi(k_x, k_y) \equiv \hat{R}(k_x, k_y) = \mathrm{FFT}\left[z(x + \Delta x, y + \Delta y) * z(x, y)\right]$
- Using convolution theorem

  $\Phi(k_x, k_y) = \hat{z}(k_x, k_y)\, \hat{z}^*(k_x, k_y) = \hat{z}^2(k_x, k_y)$
- Interpretation: energy distribution by frequencies
- Usage: signal analysis, seismology, microstructure characterization, roughness.

Spectral moments

- Spectral moment m_{pq}, $p, q \in \mathbb{N}$:

$$m_{pq} = \iint_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y)\, dk_x dk_y$$

$$m_{pq} = \left[\frac{2\pi}{L}\right]^{p+q} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} i^p j^q \; \Phi(2\pi i/L, 2\pi j/L)$$

- Generalized spectral moment m_{pq}, $p, q \in \mathbb{R}^+$
- For isotropic surface: $m_2 = m_{20} = m_{02}, \quad m_4 = 3m_{22} = m_{40} = m_{04}$
- Averaging:

$$m_2 = \frac{m_{20} + m_{02}}{2}, \quad m_4 = \frac{m_{40} + 3m_{22} + m_{04}}{3}$$

- Physical meaning:

  Height variance[1]: $m_0 = \left\langle (z - \langle z \rangle)^2 \right\rangle$

  Gradient variance: $2m_2 = \left\langle (\nabla z - \langle \nabla z \rangle)^2 \right\rangle$

  Curvature variance: $m_4 = \left\langle (\nabla \cdot \nabla z - \langle \nabla \cdot \nabla z \rangle)^2 \right\rangle$

[1] Variance is a squared standard deviation

Summary

- Fractal (self-affine) roughness
- Power spectral density (PSD)

$$\Phi(k) \sim k^{-2(H+1)}$$

  k is a wavenumber,
  H is the Hurst exponent.
- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$

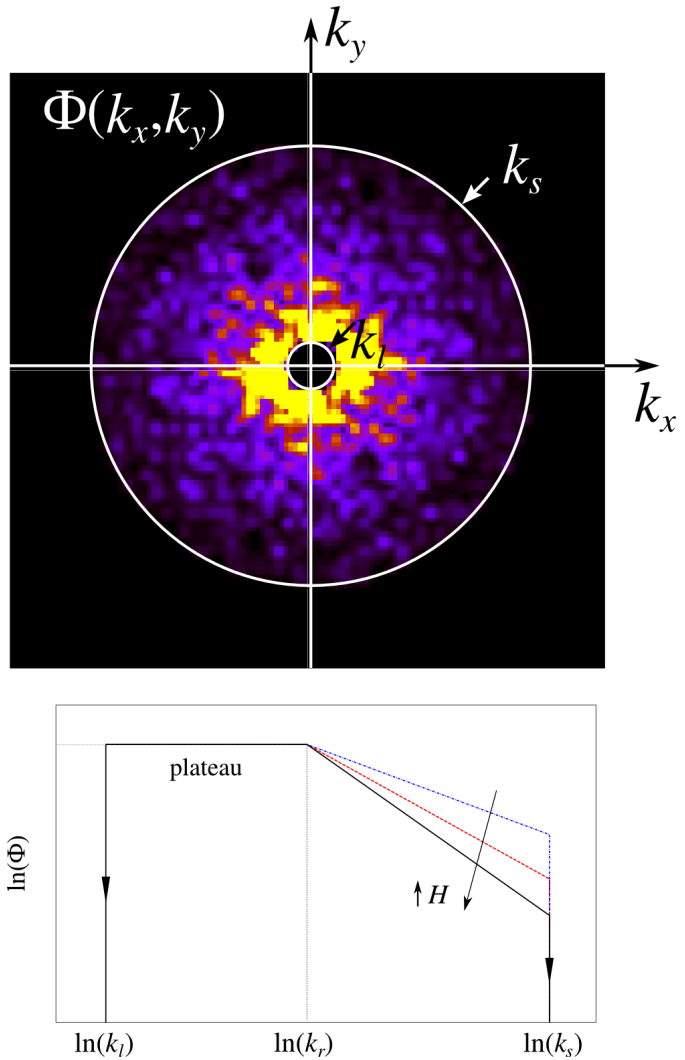


Fig. 3D and radial power spectral densities

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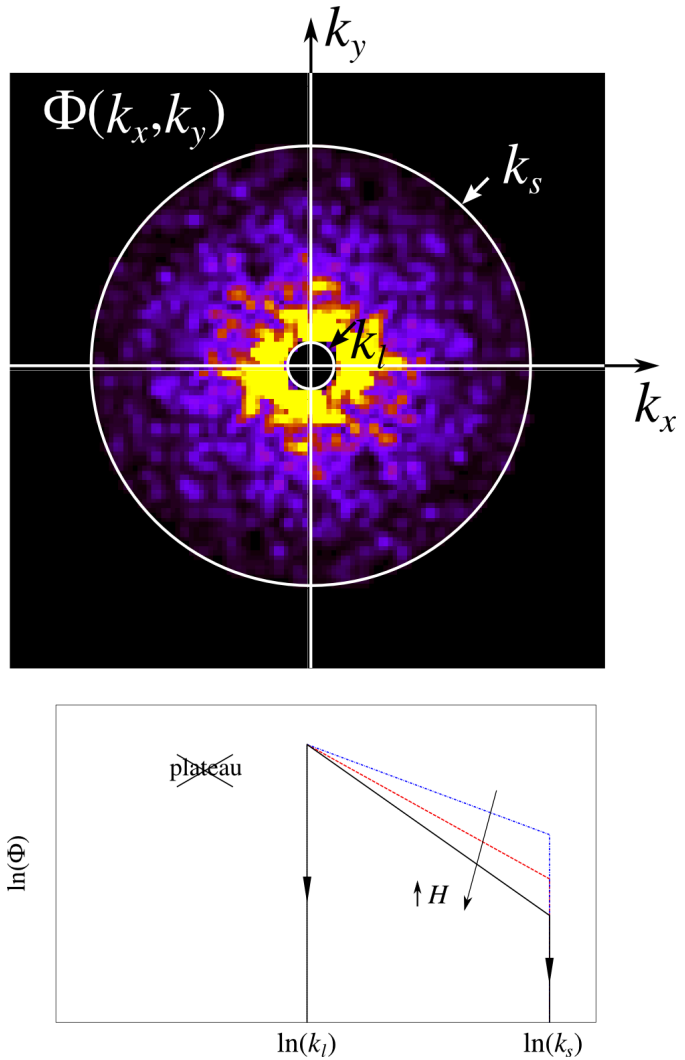

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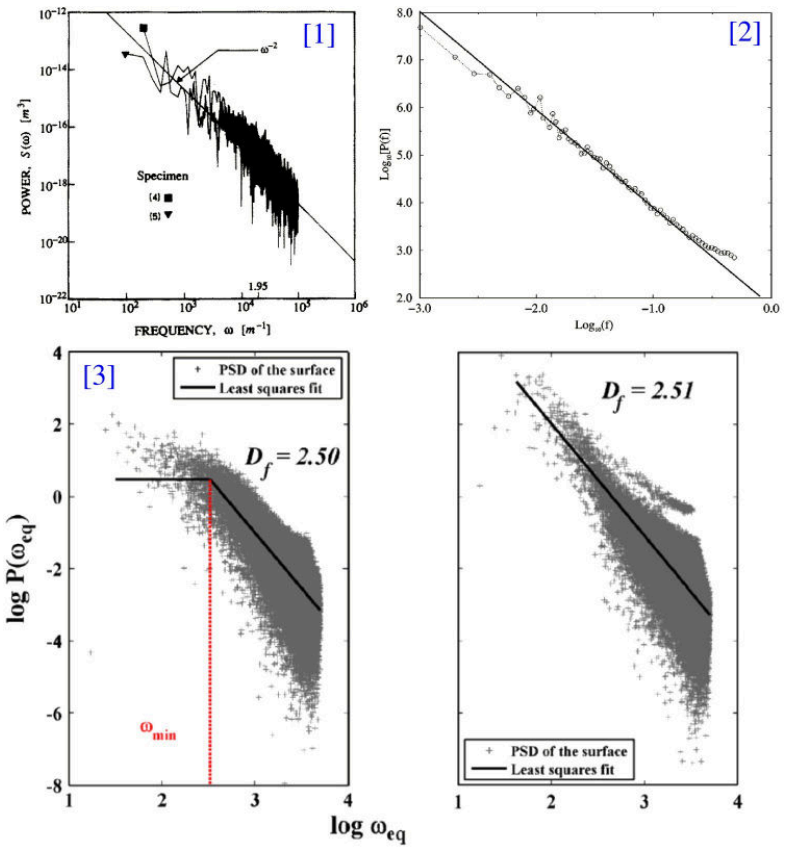

Fig. Power spectral density, measurements

[1] Majumdar, Tien, Wear 136 (1990)
[2] Schmittbuhl, Jørgen Måløy, Phys. Rev. Lett. 78 (1997)
[3] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. 42 (2009)

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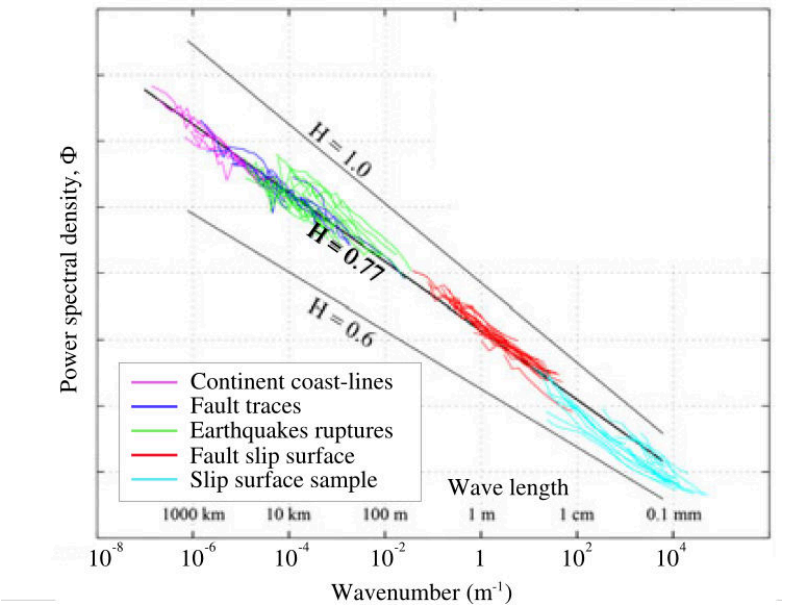

Fig. Power spectral density, geological scales

Adapted from
[4] Renard, Candela, Bouchaud, Geophys. Res. Lett. 40 (2013)

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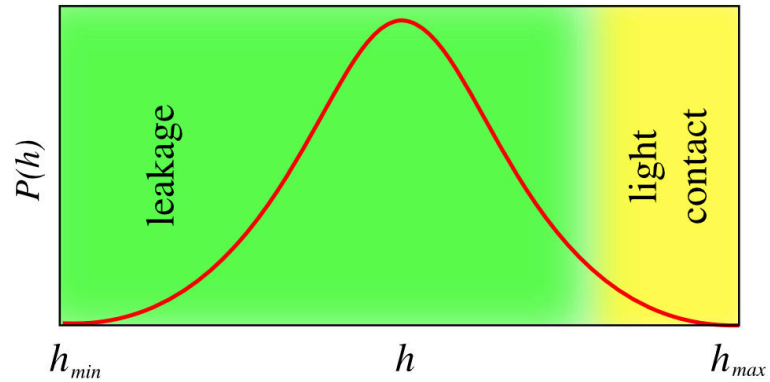

Fig. Height distribution $P(h)$

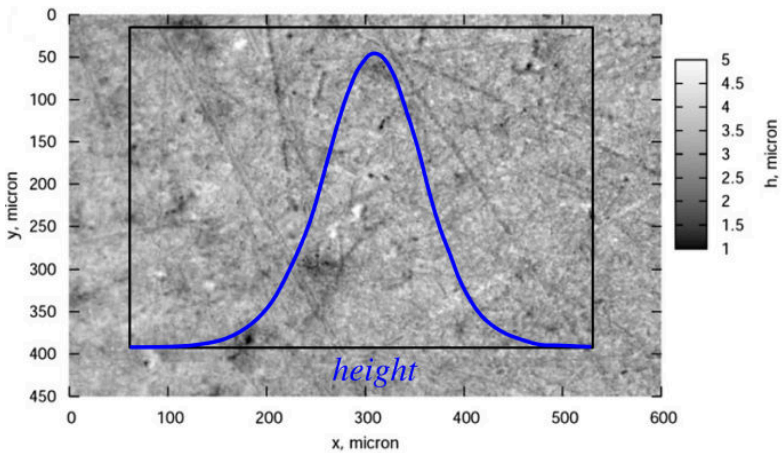

Fig. Height distribution of a polished metal surface

Summary

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- **Isotropic**/anisotropic surfaces
- **Gaussian**/non-Gaussian height distribution $P(h)$
- **Characteristics:**
  - $\sqrt{\langle z^2 \rangle}$ - *rms heights*
  - $\sqrt{\langle |\nabla z|^2 \rangle}$ - *rms slope (surface gradient)*
  - $\alpha = m_{00} m_{40} / m_{20}^2$ - *breadth of the spectrum (Nayak's parameter[B])*,

spectral moments $m_{pq} = \iint_{-\infty}^{\infty} k_x^p k_y^q \Phi(k_x, k_y)\, dk_x dk_y$

- **Random process theory**

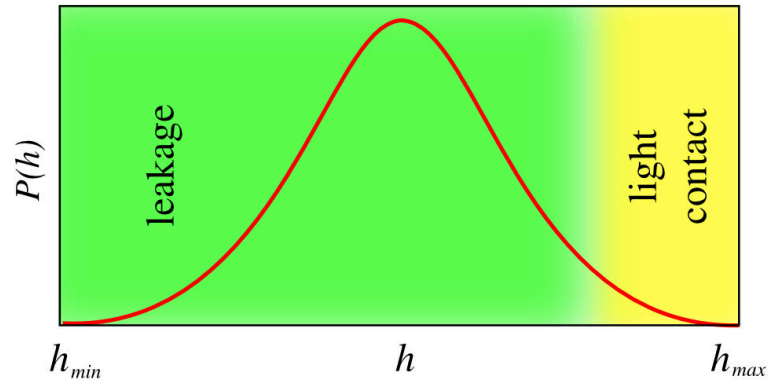

Fig. Height distribution $P(h)$

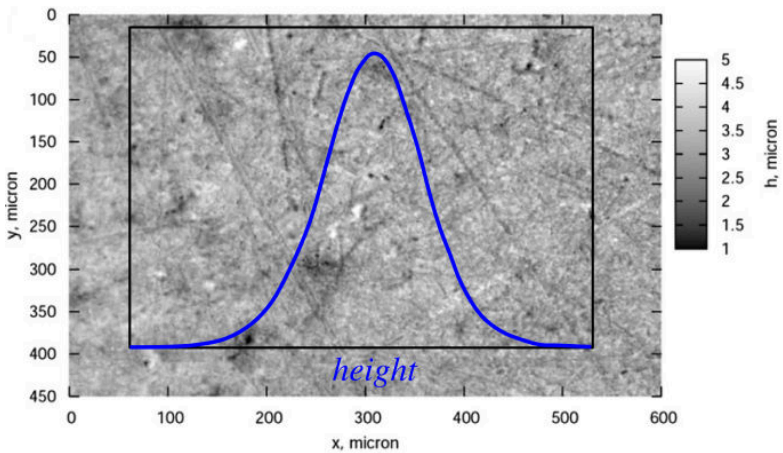

Fig. Height distribution of a polished metal surface

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
[D] Borri, Paggi, J. Phys. D Appl Phys 48:045301 (2015)

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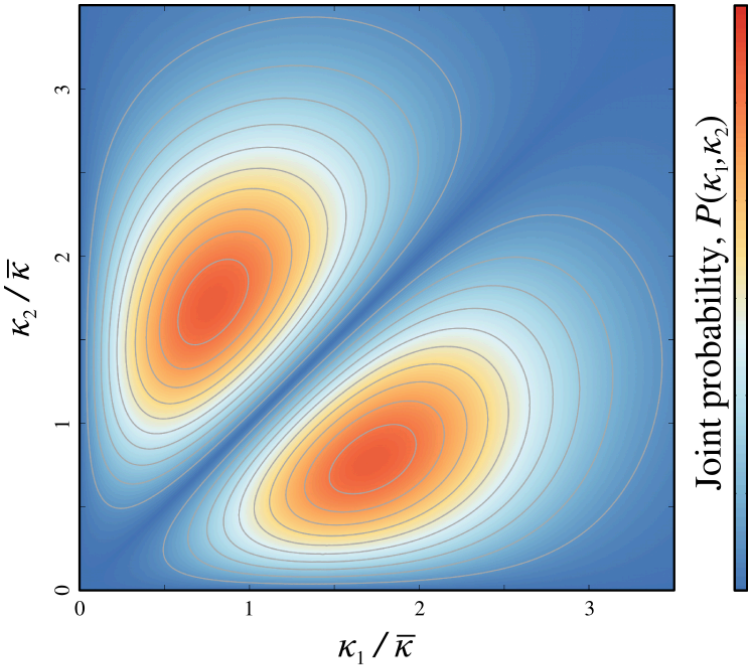

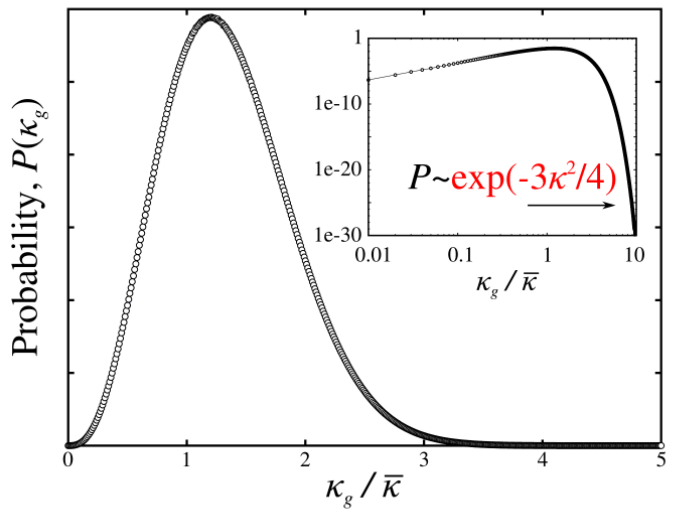

$P \sim \kappa^2 \exp(-3\kappa^2/4)\ \mathrm{erf}(3\kappa/2)$

Distribution of asperity curvatures

[A] Longuet-Higgins, Philos. Trans. R. Soc. A 250:157 (1957)
[B] Nayak, J. Lub. Tech. (ASME) 93:398 (1973)
[C] Greenwood, Wear 261: 191 (2006)
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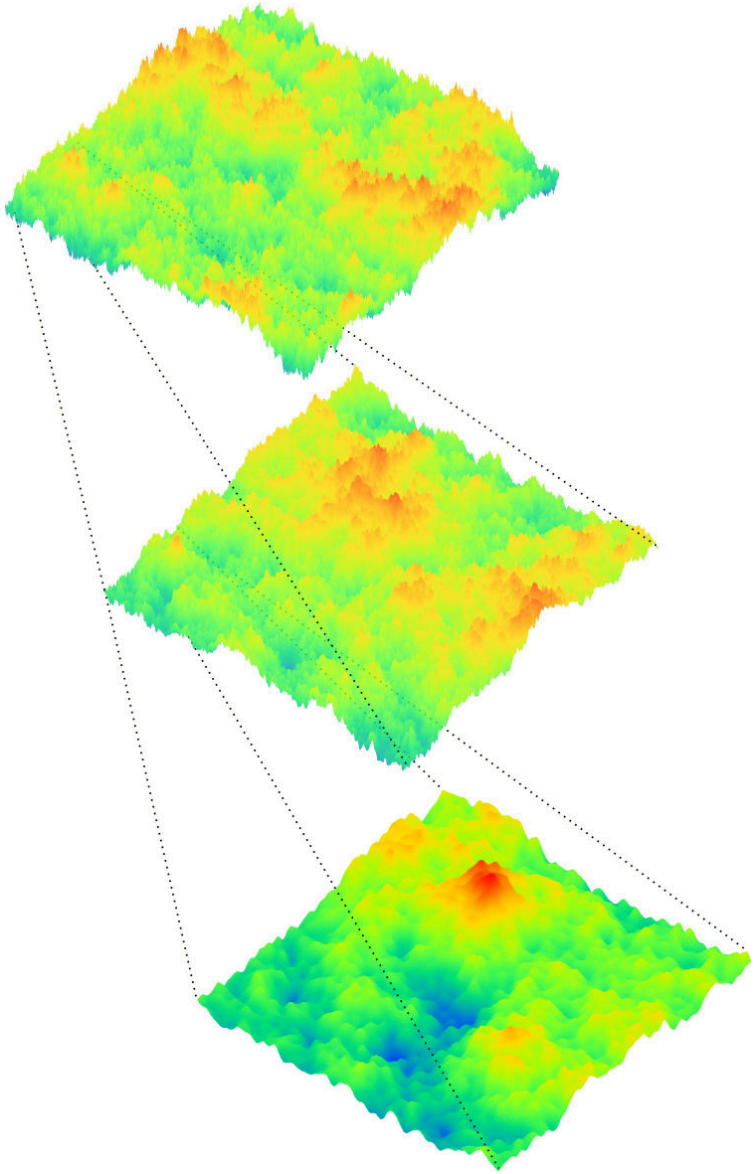


Fig. Example of a rough surface for $H = 0.3$

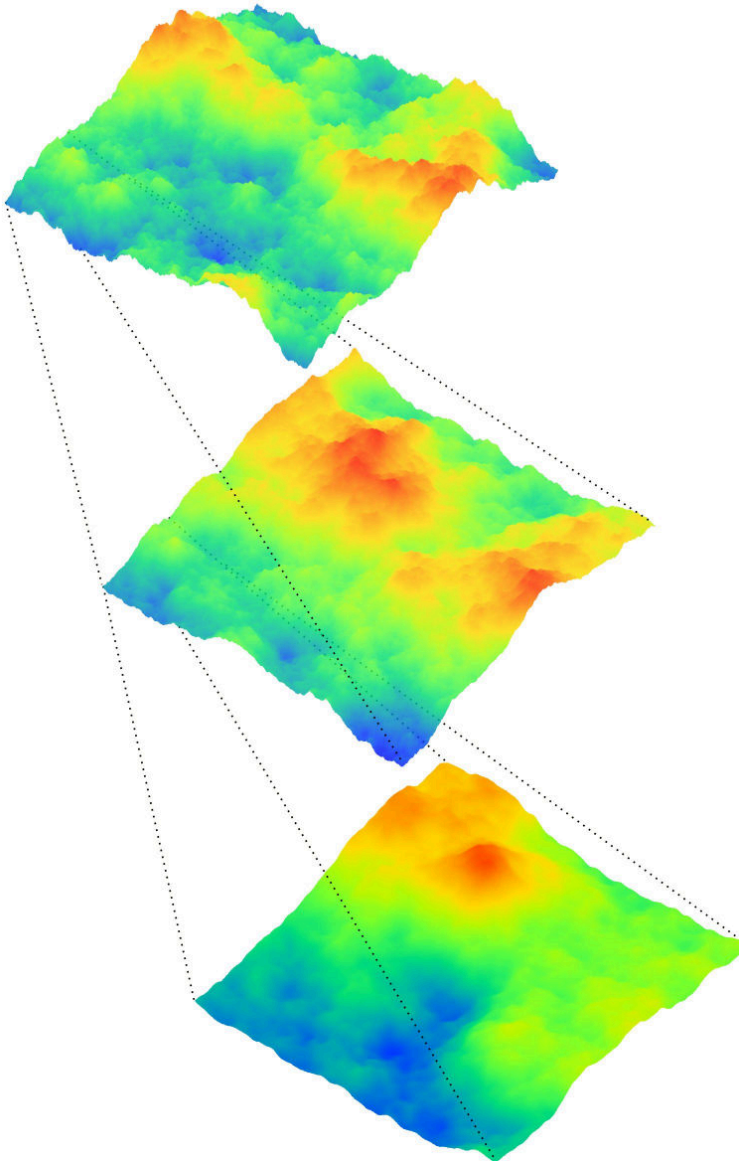
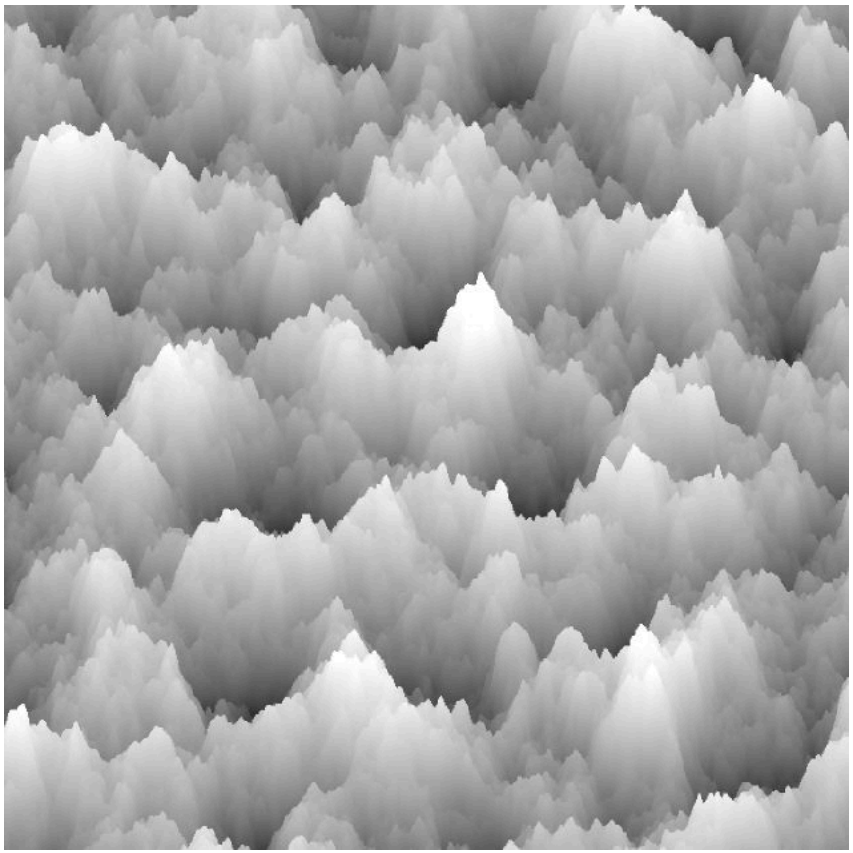


Fig. Example of a rough surface for $H = 0.8$

Recall: the Hurst exponent H and the fractal dimension D in 2D space are interconnected via $D = 3 - H$

Fractals



Flight over a rough surface

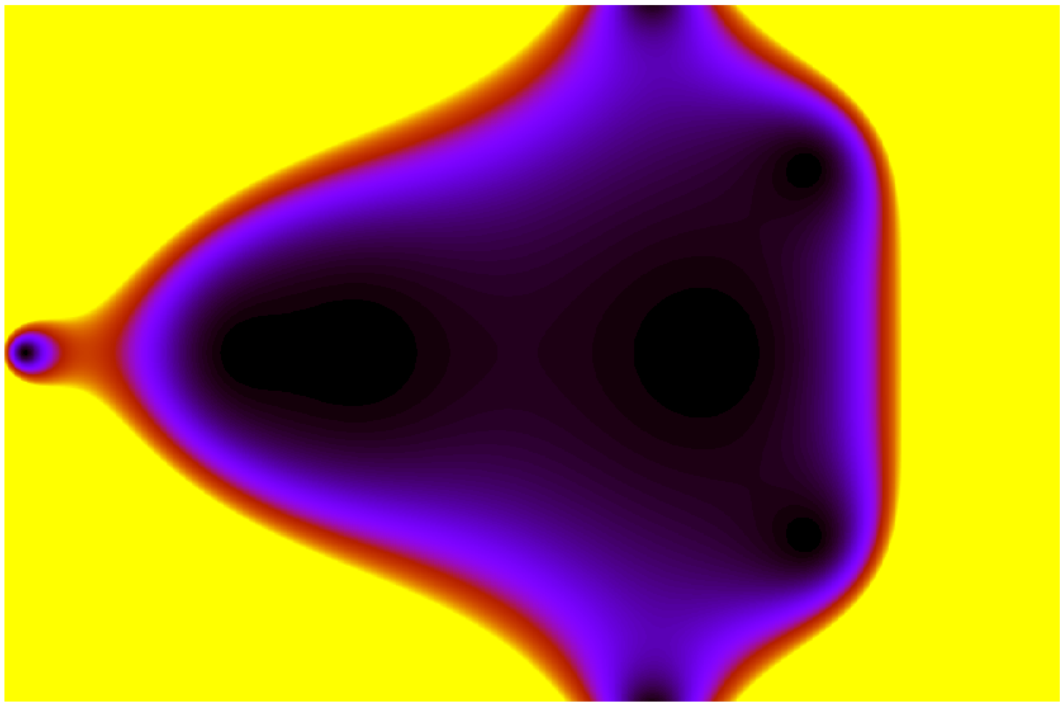
Fractals

Romanesco broccoli www.fourmilab.ch

Fractals

- Mandelbrot set (not a fractal)
- Recursive function

$$z_{i+1} = z_i^2 + z, \quad z \in \mathbb{C}$$



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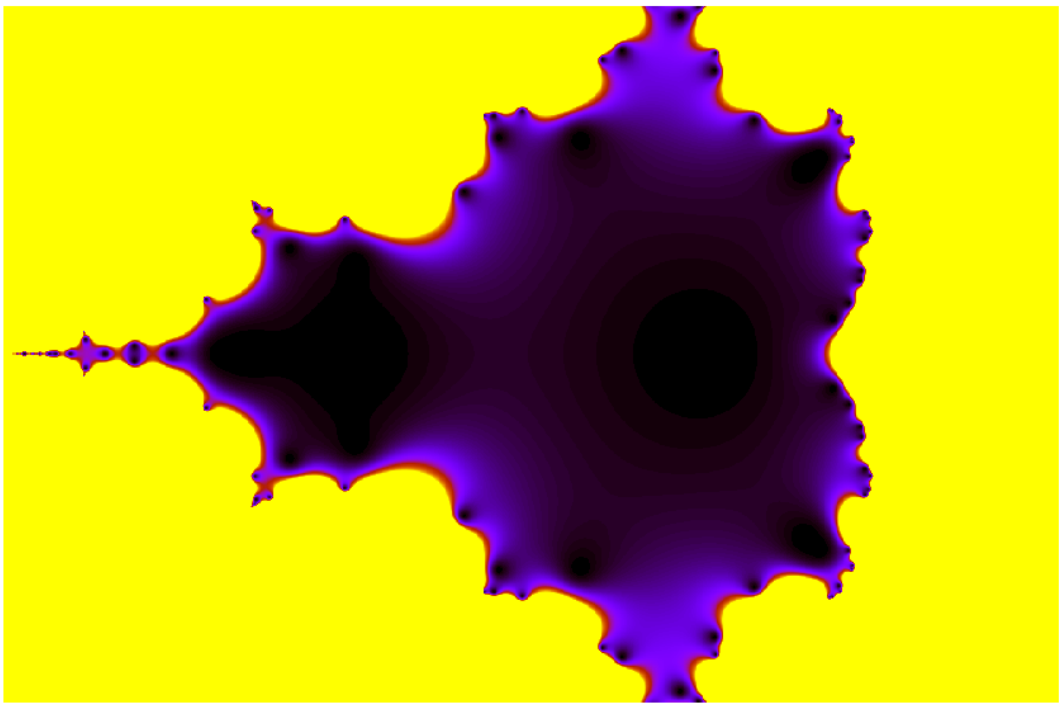

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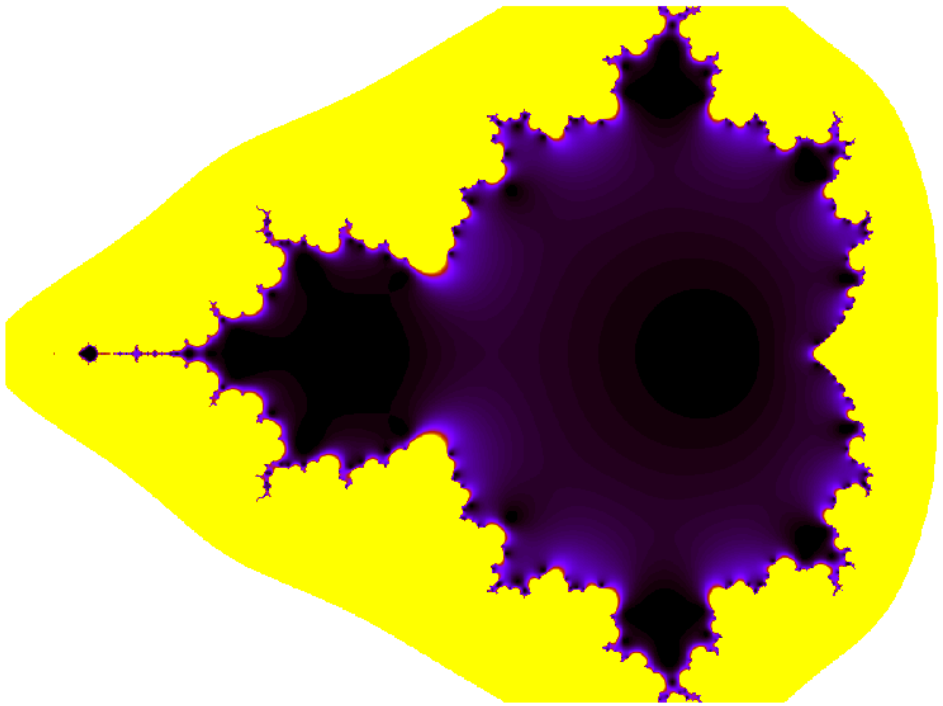

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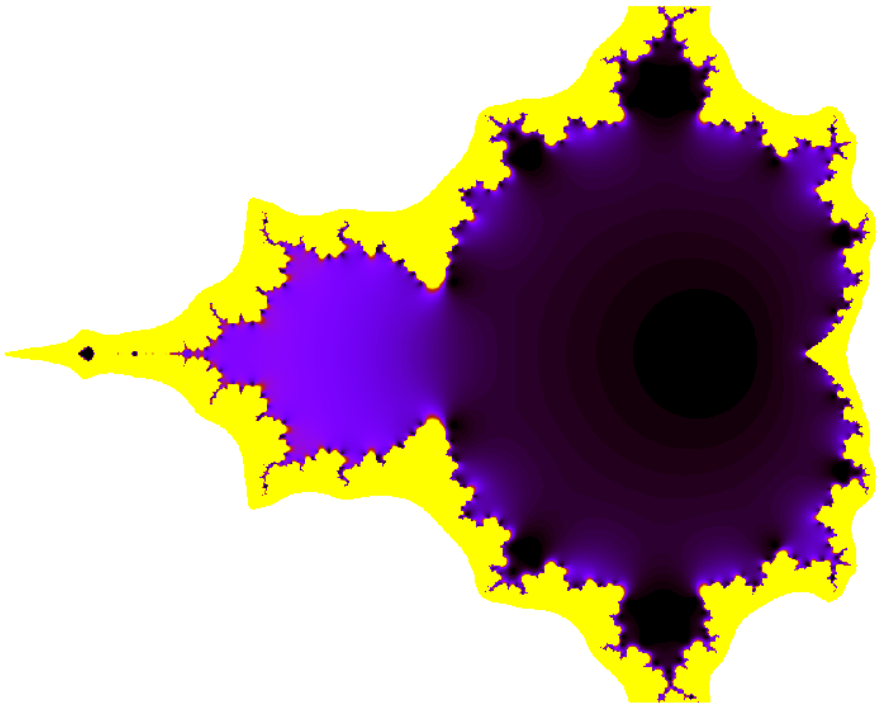

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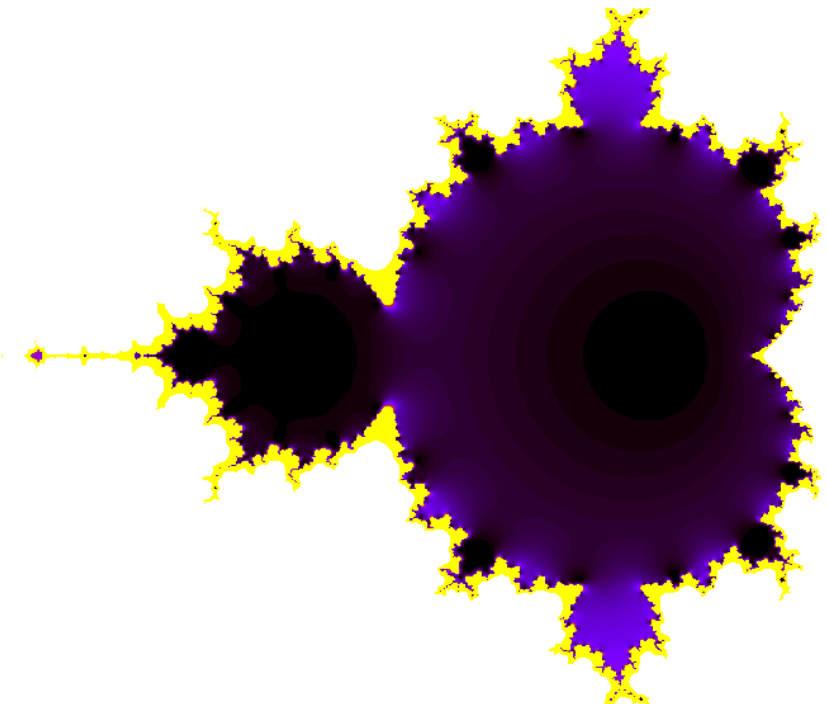

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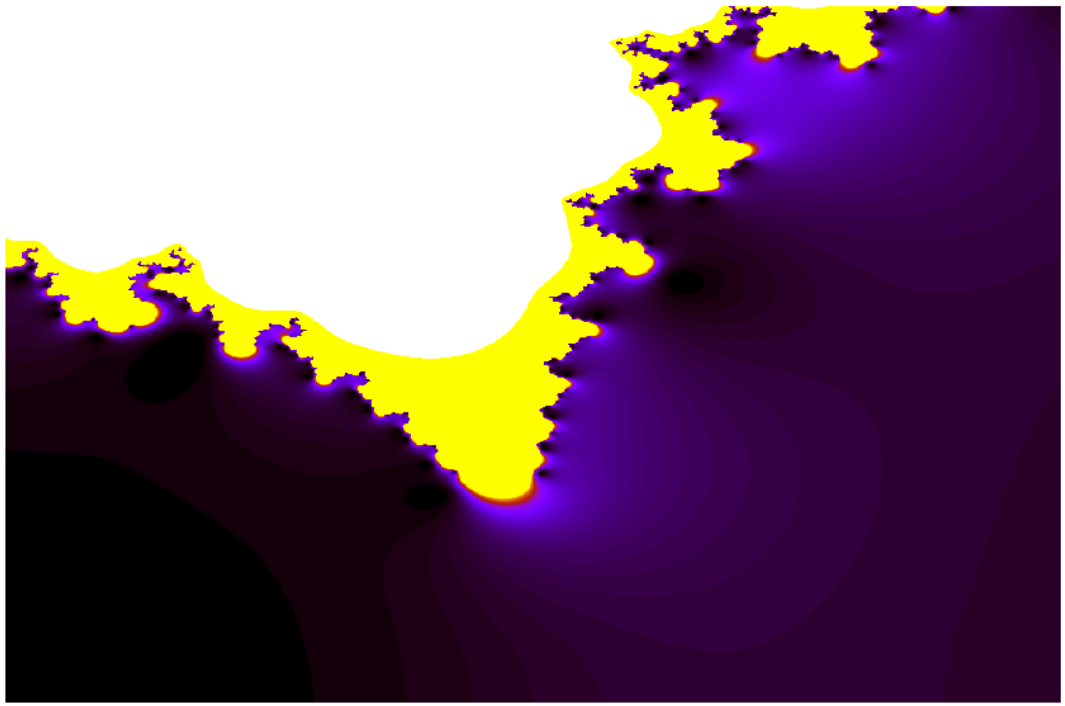

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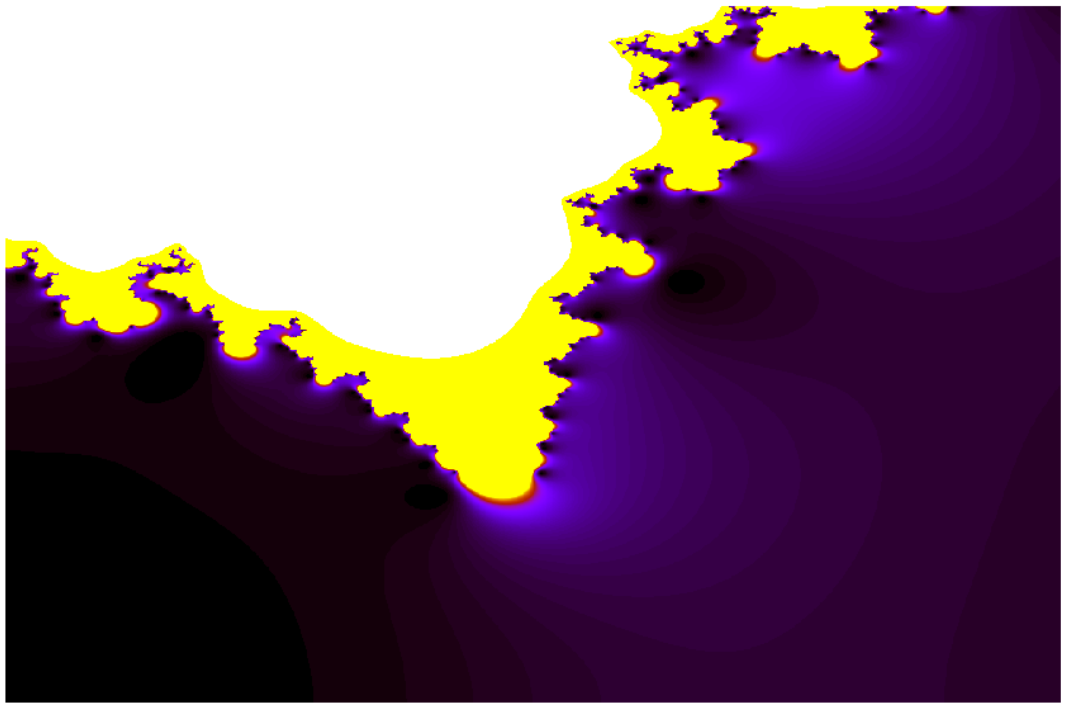

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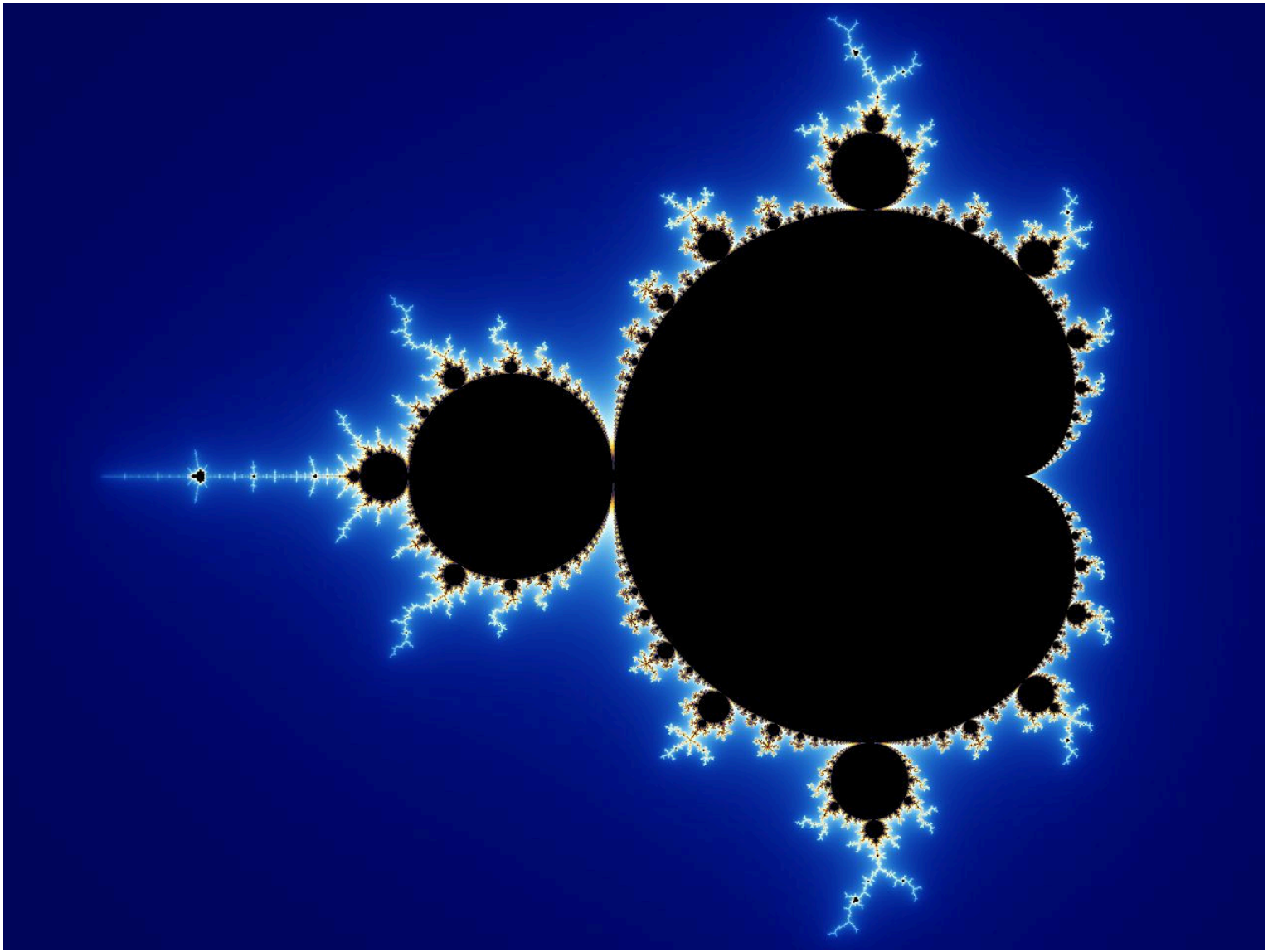
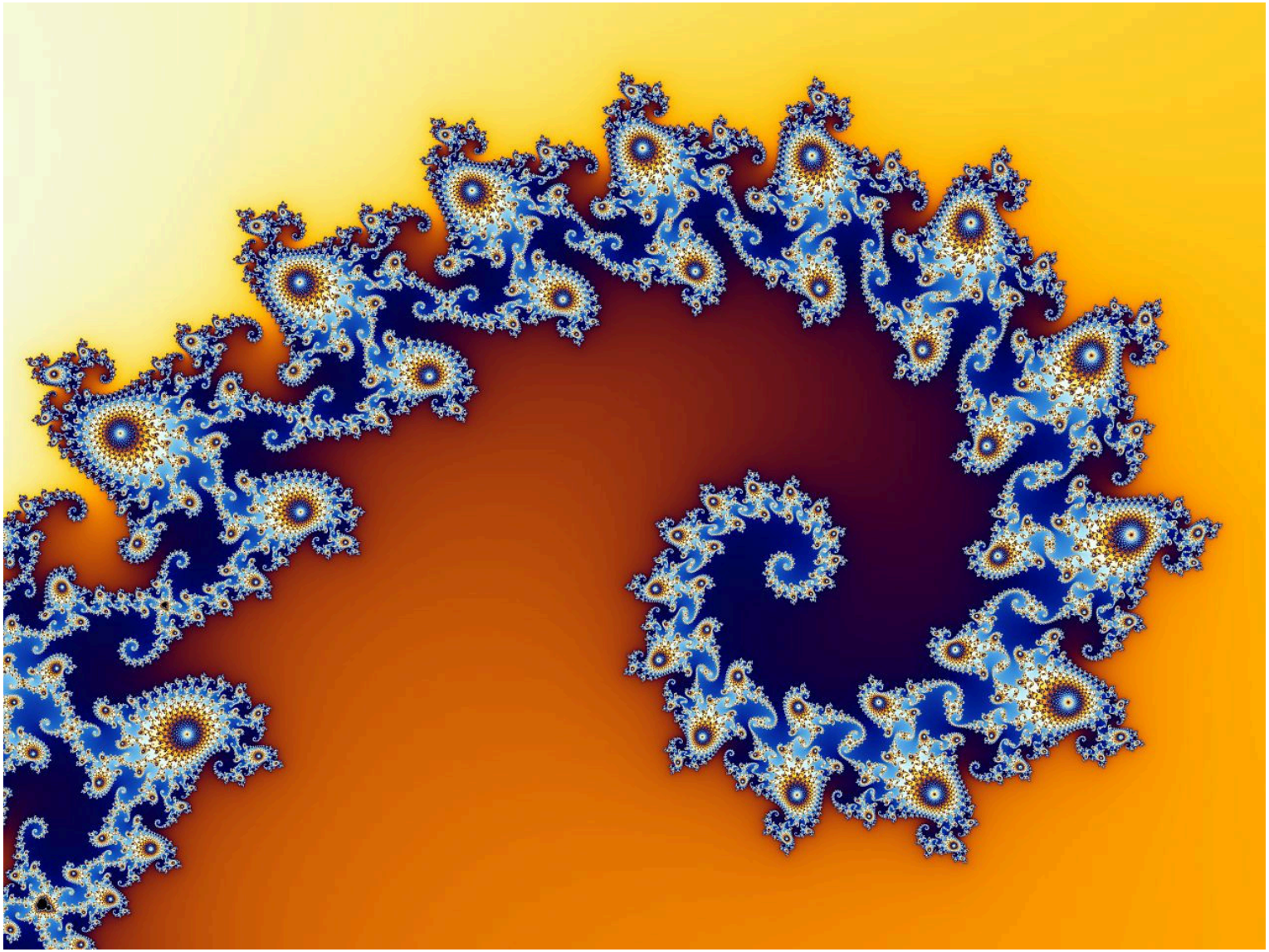


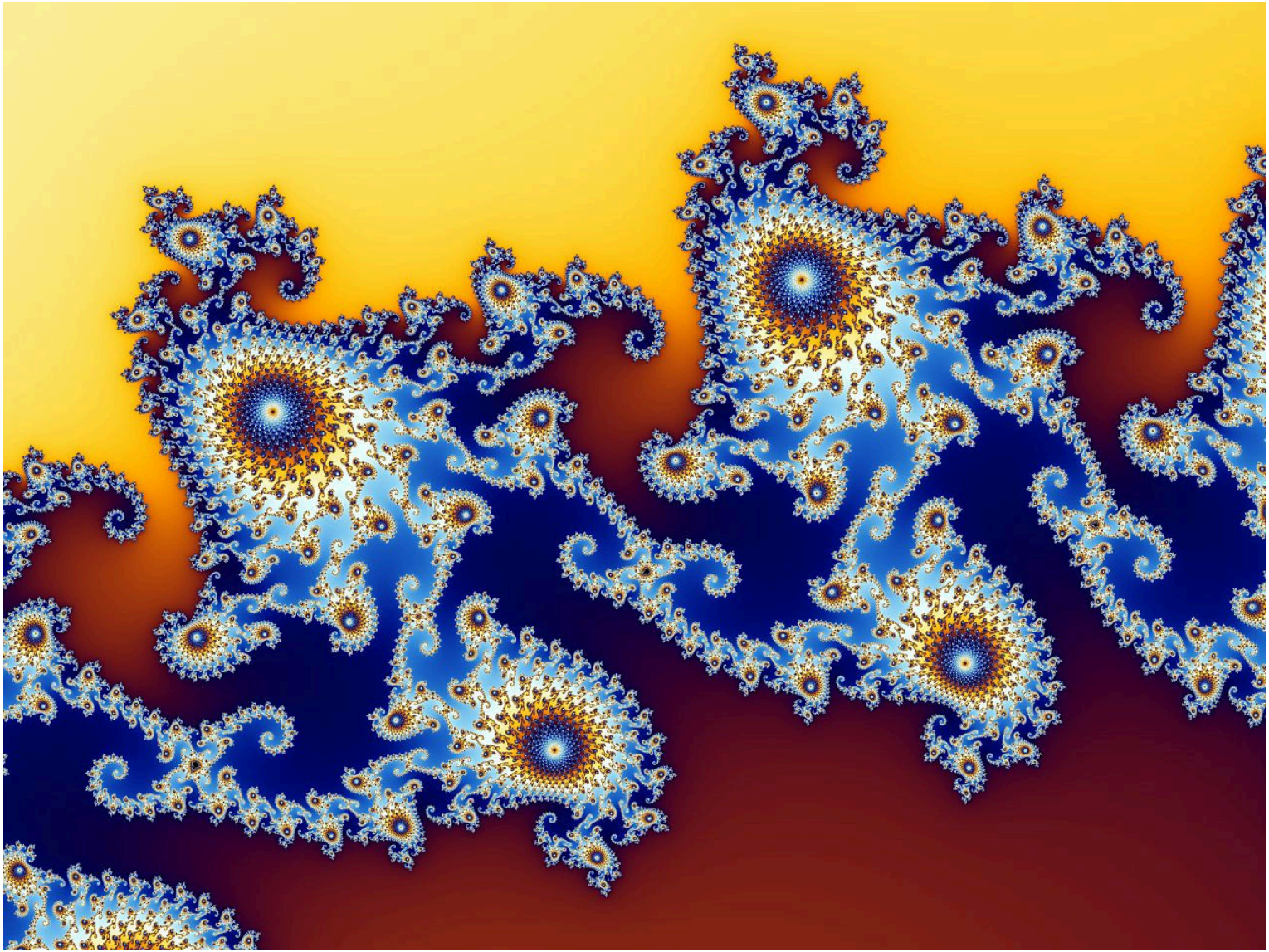
Mandelbrot set (Wikipedia)

Fractals



Element of Mandelbrot set (Wikipedia)

Fractals



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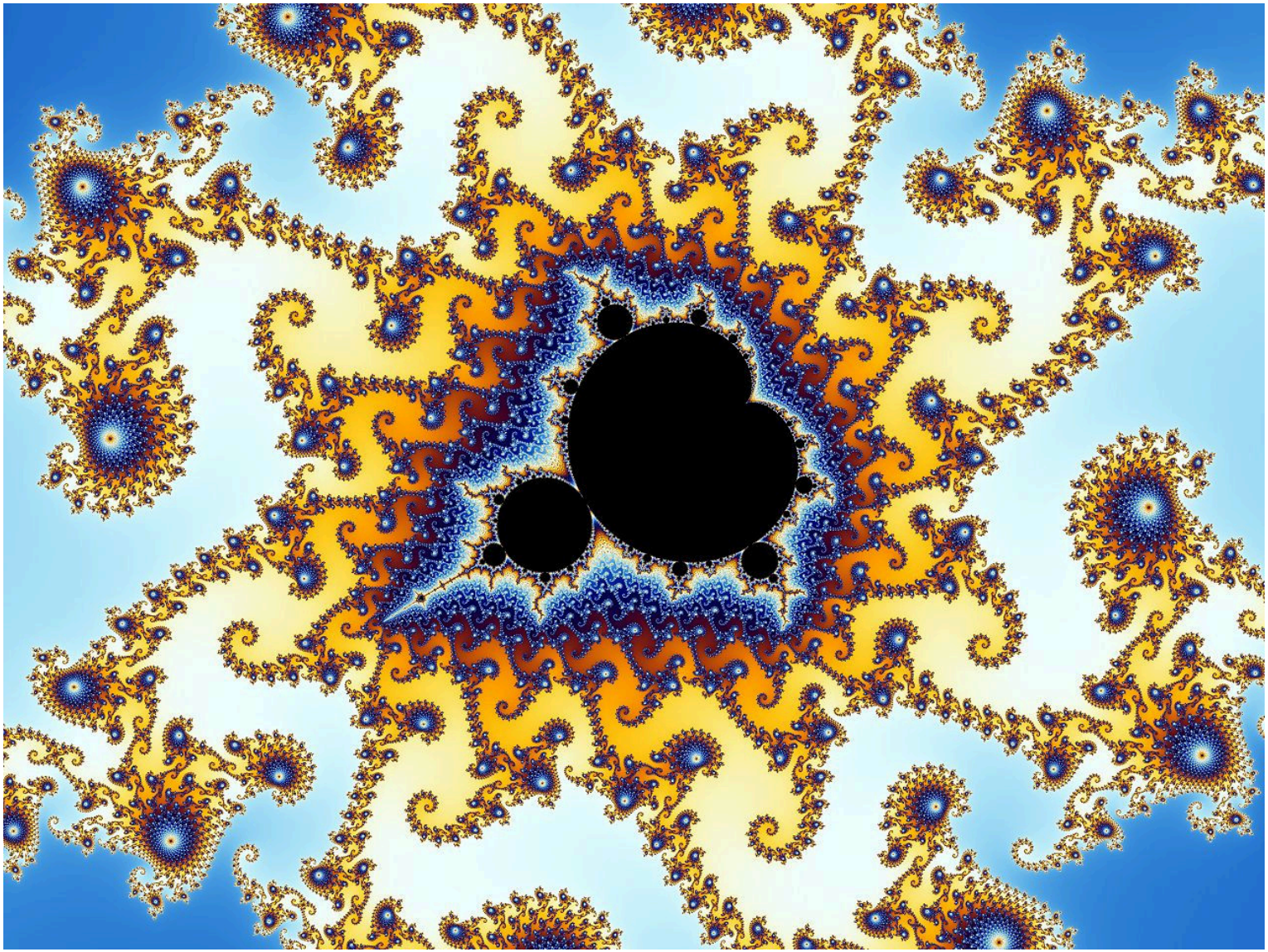

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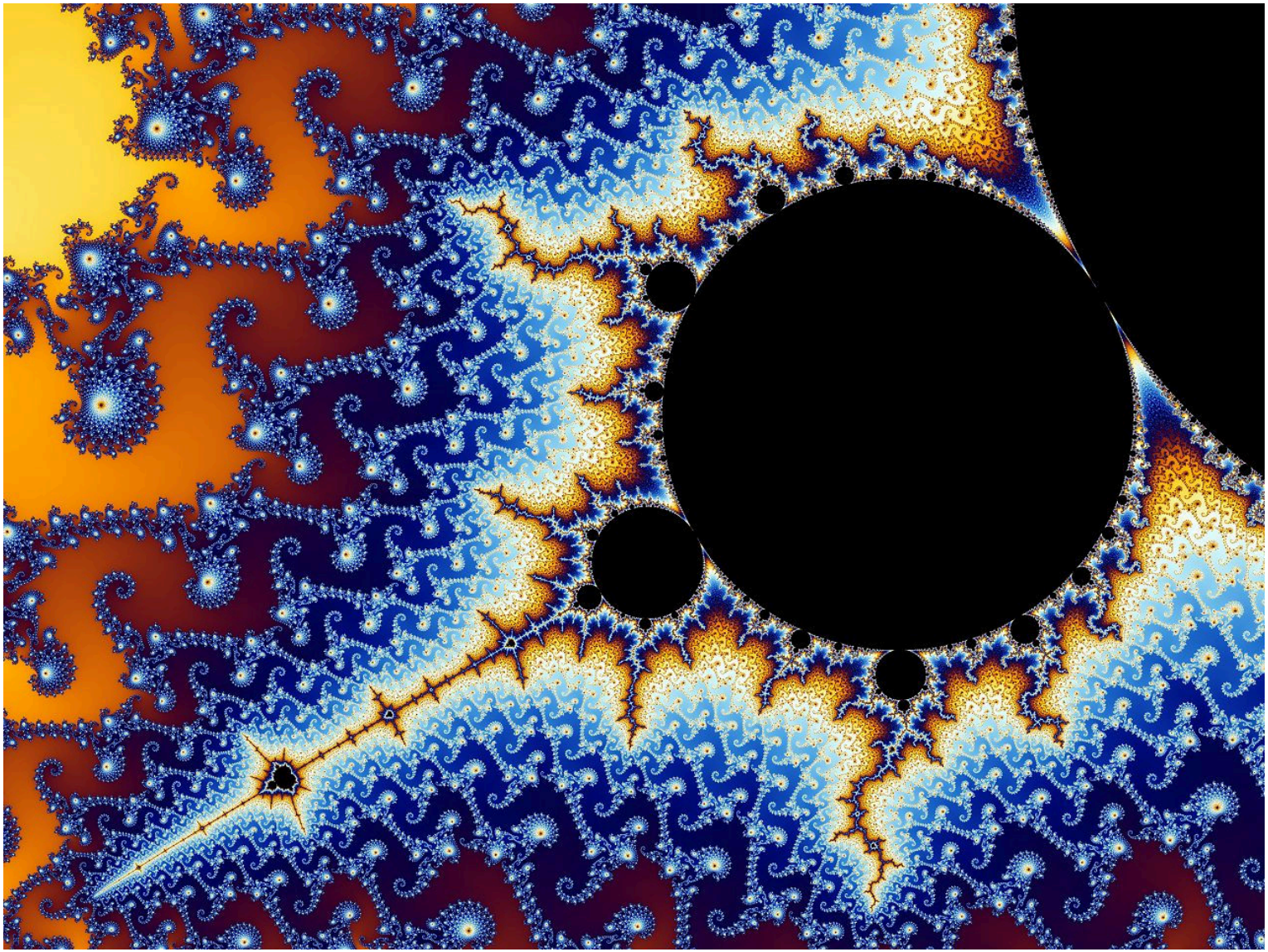

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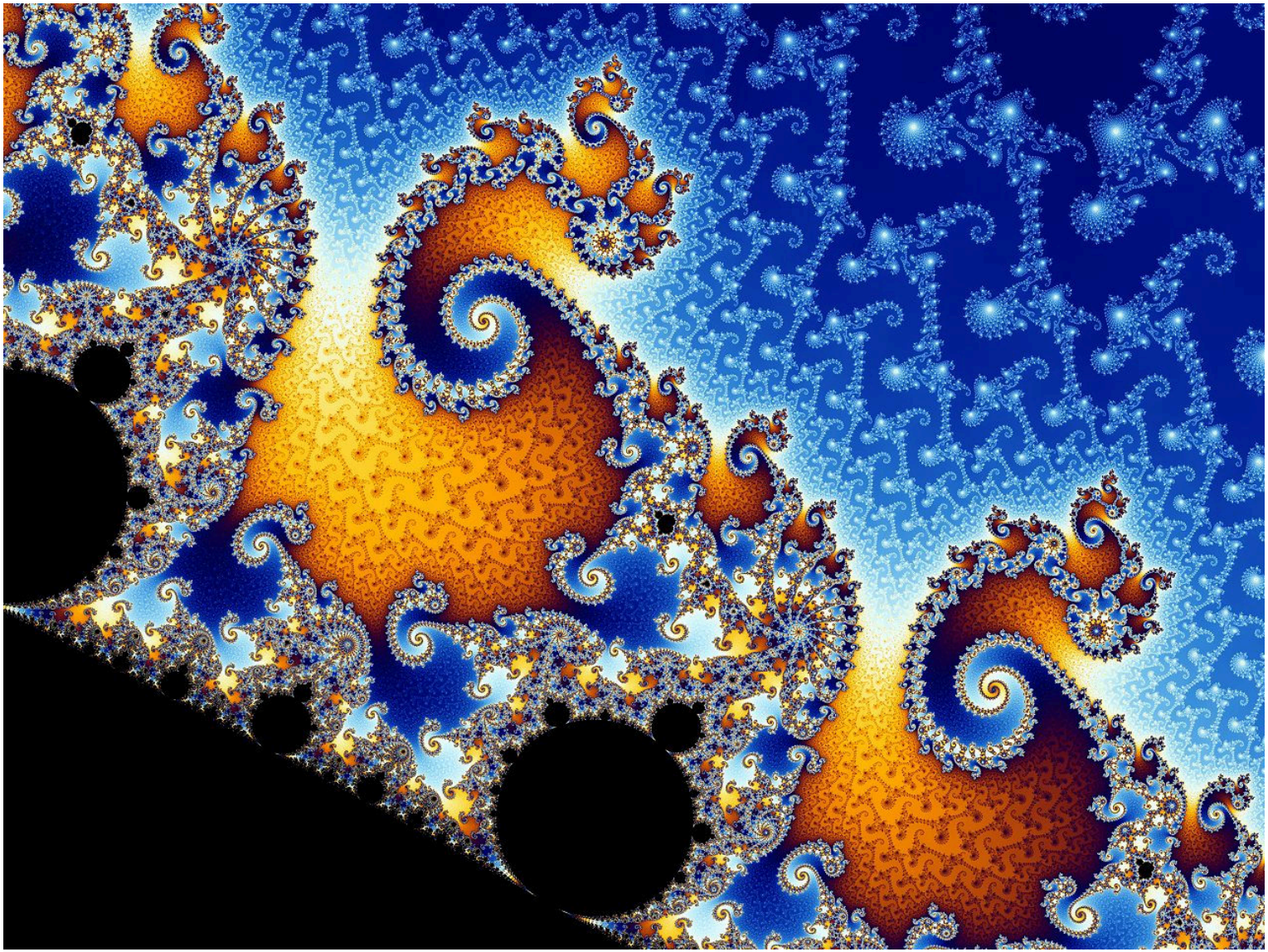
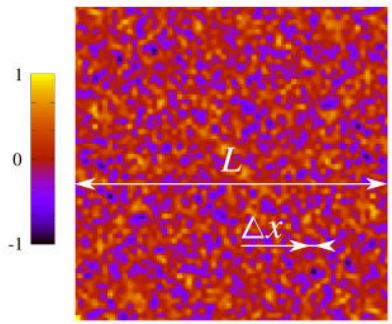


Element of Mandelbrot set (Wikipedia)

Animation

Synthesized rough surfaces: in pictures

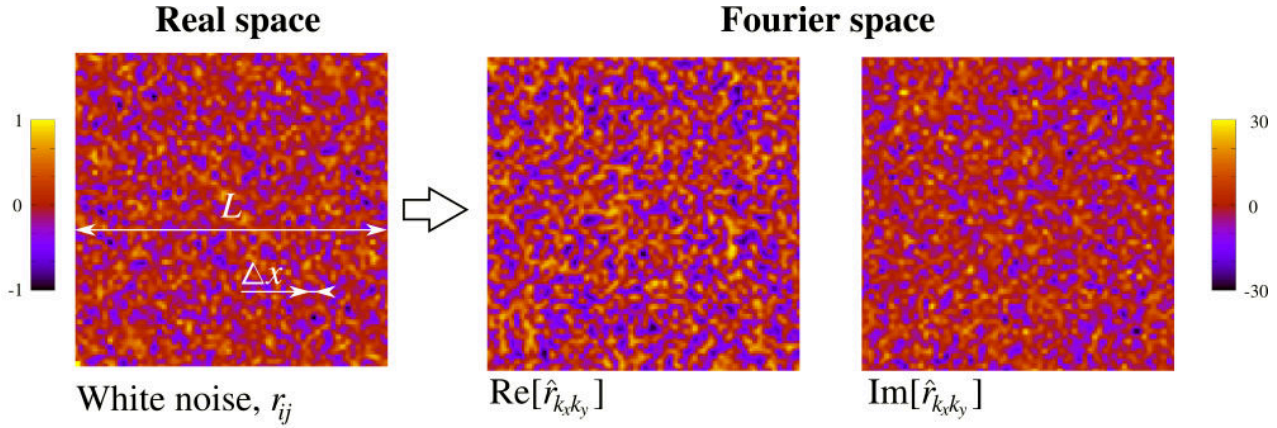
Real space **Fourier space**



White noise, r_{ij}

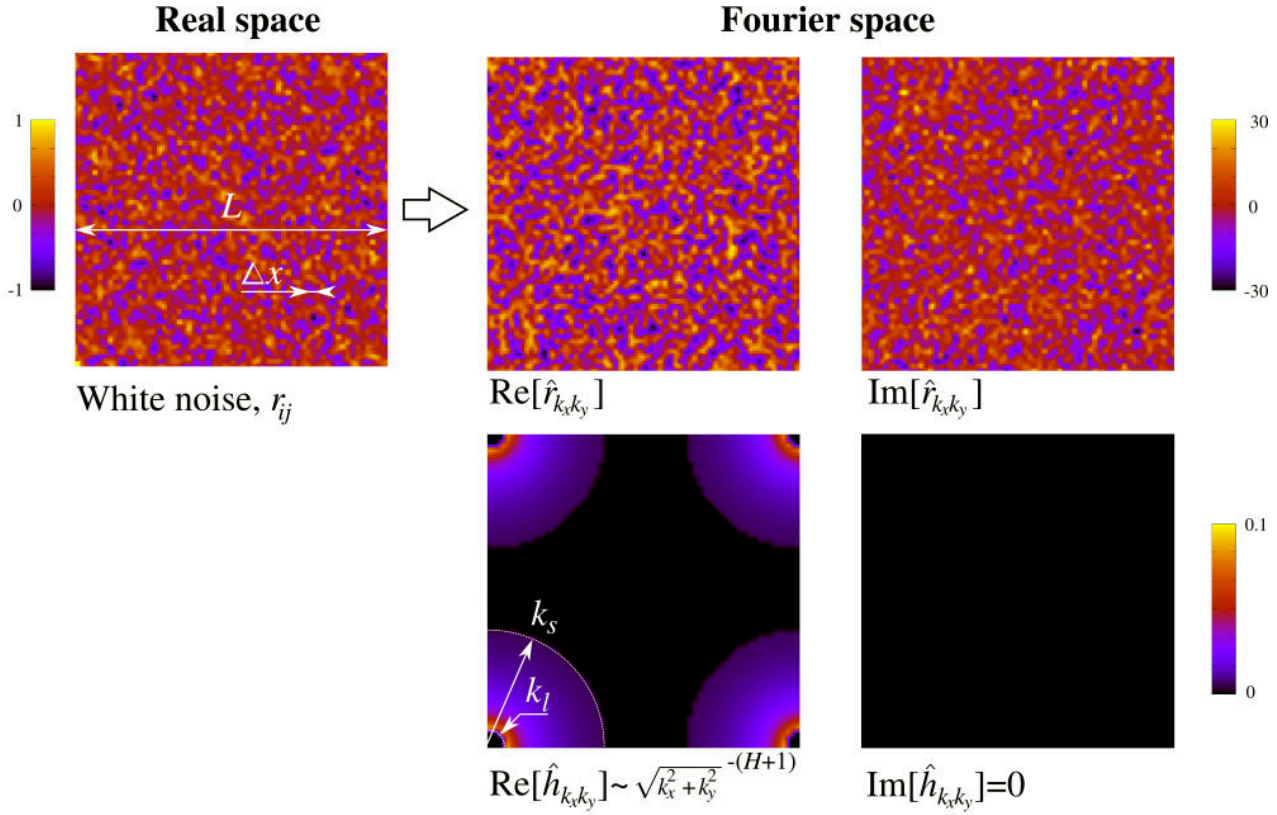
[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



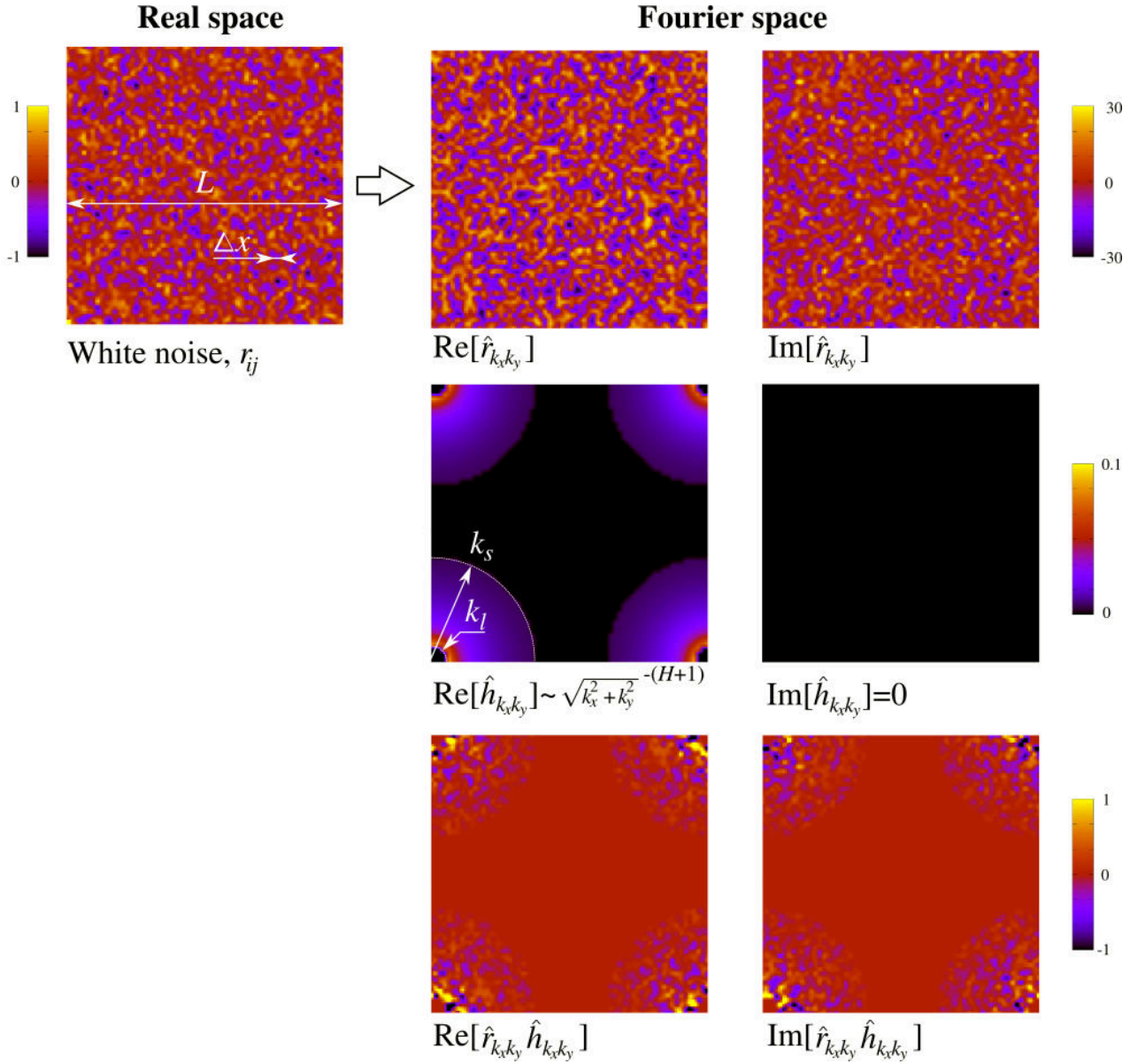
[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



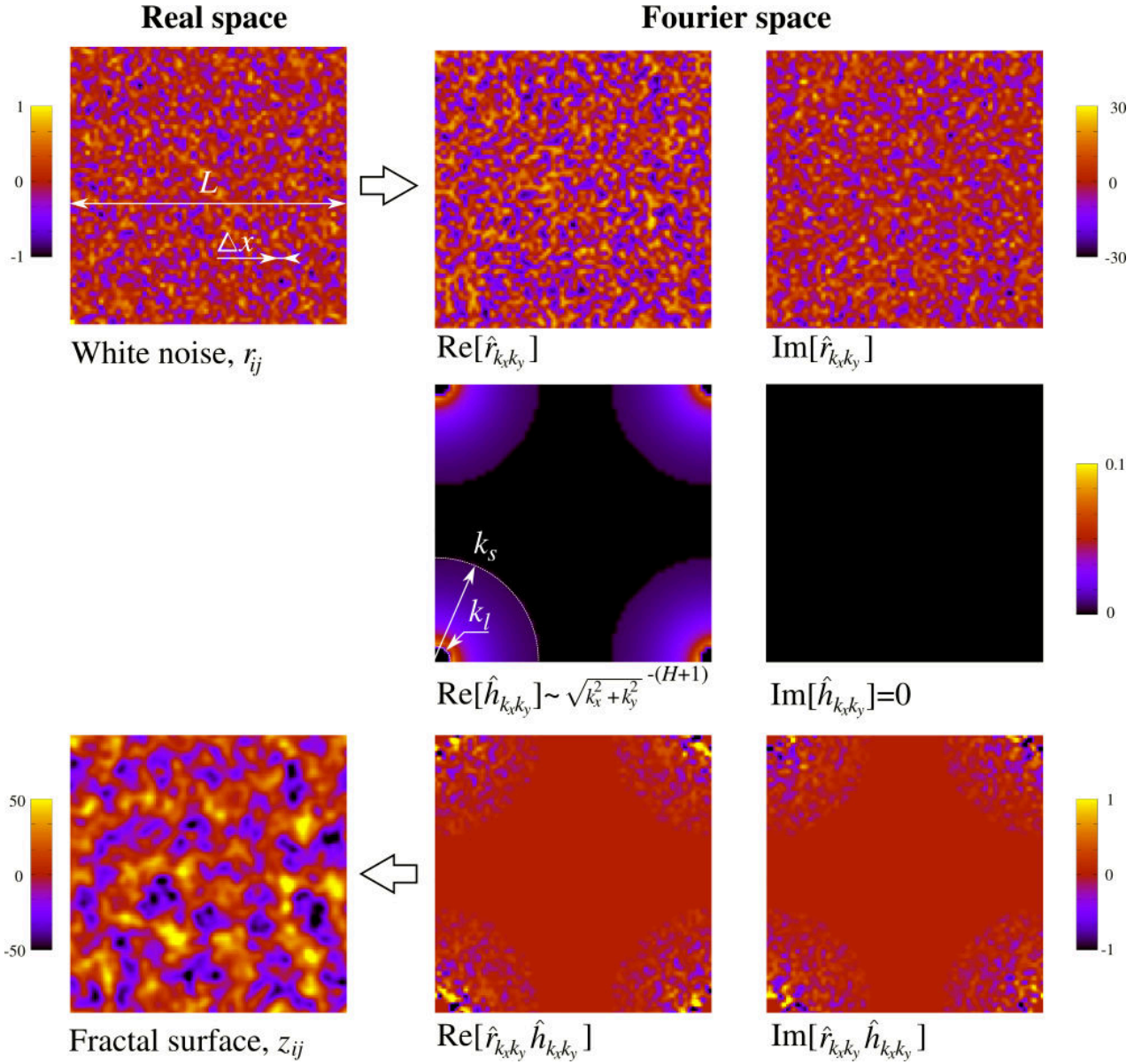
[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in pictures



[1] Y. Z. Hu and K. Tonder, Int. J. Machine Tools Manuf. 32, 83 (1992)

Synthesized rough surfaces: in equations

- White noise:
  $w(x_i, y_j), \quad \langle w \rangle = 0, \quad \langle w^2 \rangle = \Phi_0$
- Transform in Fourier space:
  $$\hat{w}_{ij} = \hat{w}(k_x, k_y) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} w(x_i, y_i) \exp[-\mathrm{i}(k_x x_i + k_y y_j)], \quad \langle \hat{w} \hat{w}^* \rangle = \langle w^2 \rangle = \Phi_0$$
- Create a filter
  $$\hat{f}_{ij} = \hat{f}(k_x, k_y) = \begin{cases} \left[\frac{K_x^2 + K_y^2}{k_l^2} \right]^{-(1+H)/2}, & \text{for } 1 \le \frac{\sqrt{K_x^2 + K_y^2}}{k_l} \le \zeta \\ 0, & \text{elsewhere,} \end{cases},$$
  where $K_x = (s+1)\pi/L - sk_x$, $K_y = (t+1)\pi/L - tk_y$ for $s, t \in \{-1, 1\}$, $\zeta = k_s/k_l$
- Filter white noise:
  $$\hat{z}_{ij} = \hat{z}(k_x, k_y) = \Re(\hat{f}_{ij}) \left[\Re(\hat{w}_{ij}) + \mathrm{i}\, \Im(\hat{w}_{ij}) \right]$$
- Back to real space:
  $$z(x_i, y_j) = \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \hat{z}_{lm} \exp[\mathrm{i}2\pi(lx_i + my_j)/L]$$

Synthesized rough surfaces: in equations II

- Power spectral density:

  $\Phi(k_x, k_y) = \hat{z}(k_x, k_y)\hat{z}^*(k_x, k_y) = \hat{f}^2(k_x, k_y)\hat{w}^2(k_x, k_y)$

- Averaging over multiple samples:

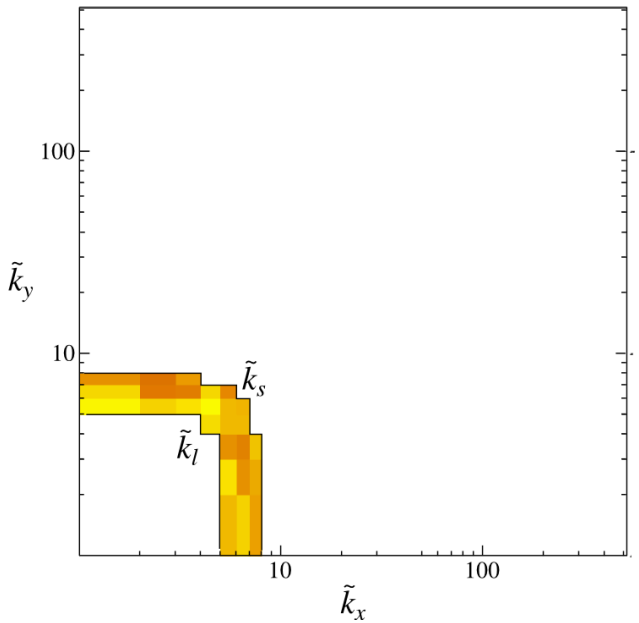
$$\langle \Phi(k_x, k_y) \rangle = \langle \hat{w}^2(k_x, k_y) \rangle \hat{f}^2(k_x, k_y) = \begin{cases} \Phi_0 \left[\frac{\sqrt{K_x^2 + K_y^2}}{k_l} \right]^{-2(1+H)}, & \text{for } 1 \leq \frac{\sqrt{K_x^2 + K_y^2}}{k_l} \leq \zeta \\ 0, \text{ elsewhere,} \end{cases}$$

- For isotropic surface:

$$\langle \Phi(K) \rangle = \begin{cases} \Phi_0 (K/k_l)^{-2(1+H)}, & \text{if } 1 \leq K/k_l \leq \zeta \\ 0, & \text{otherwise.} \end{cases}$$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



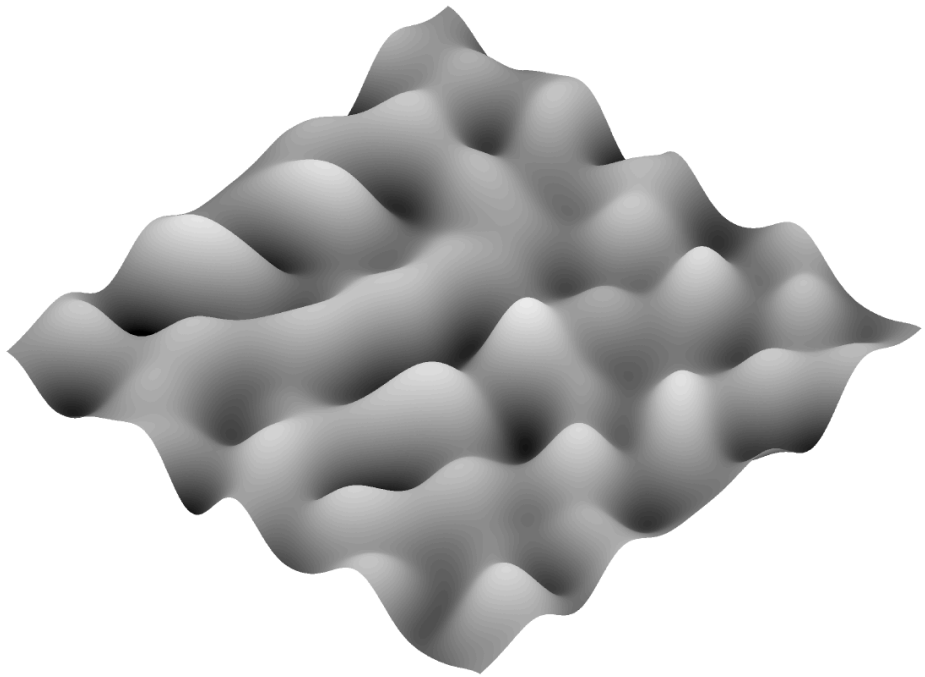
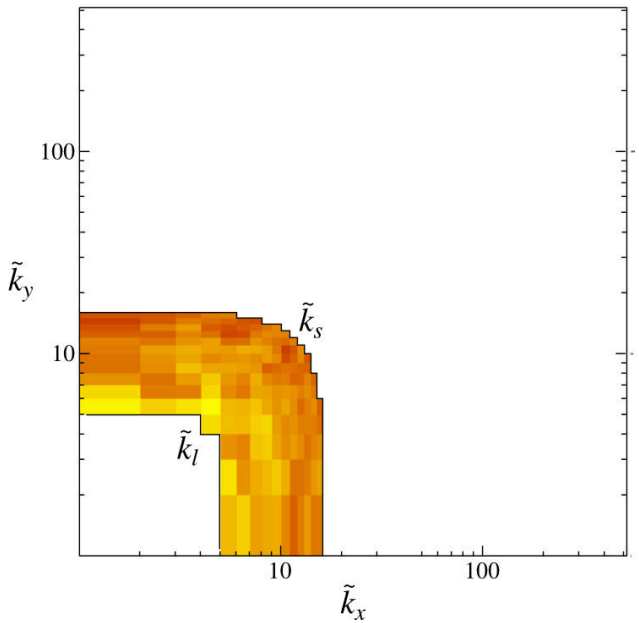


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 8$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



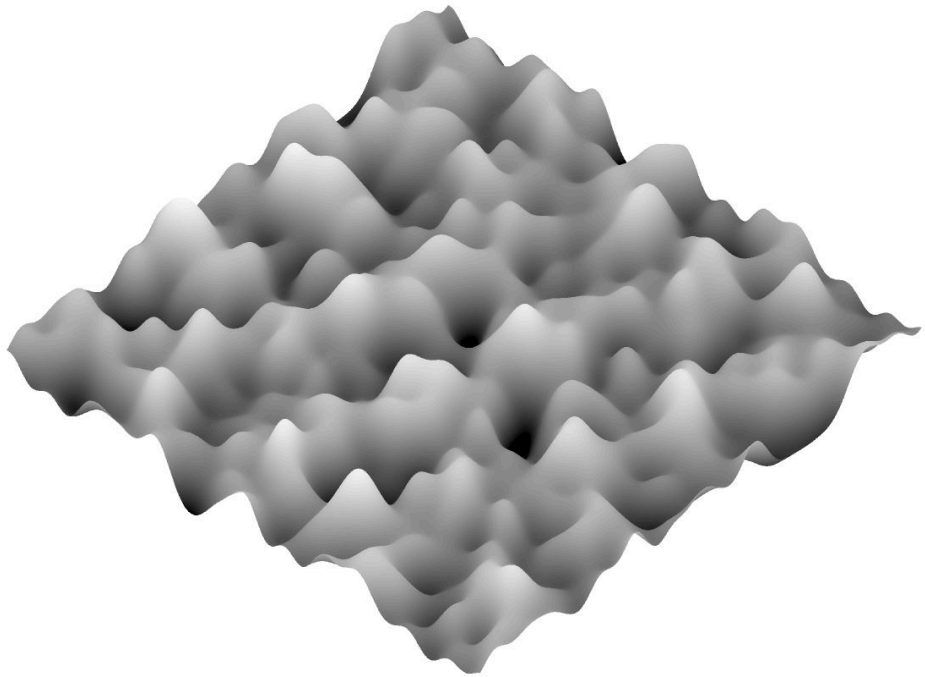
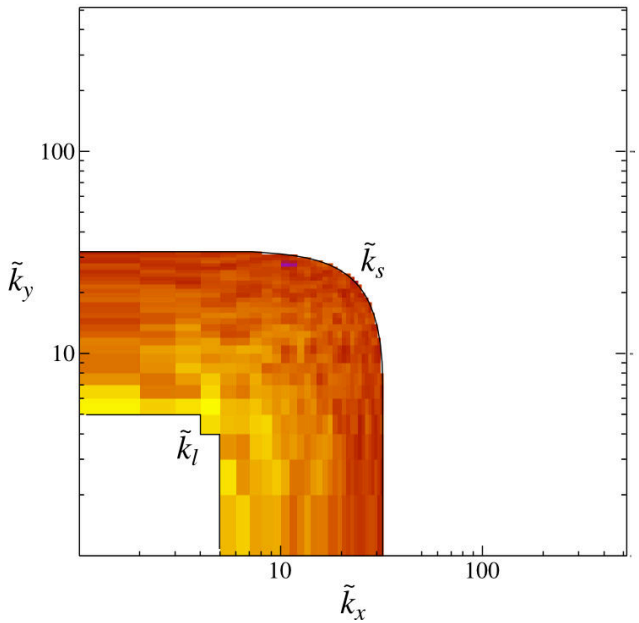


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 16$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



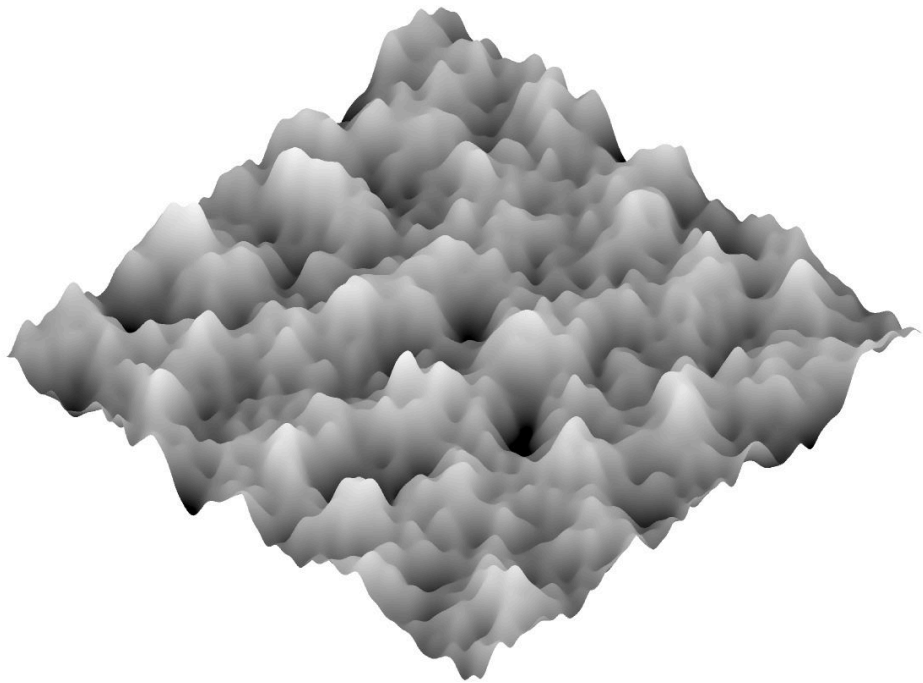
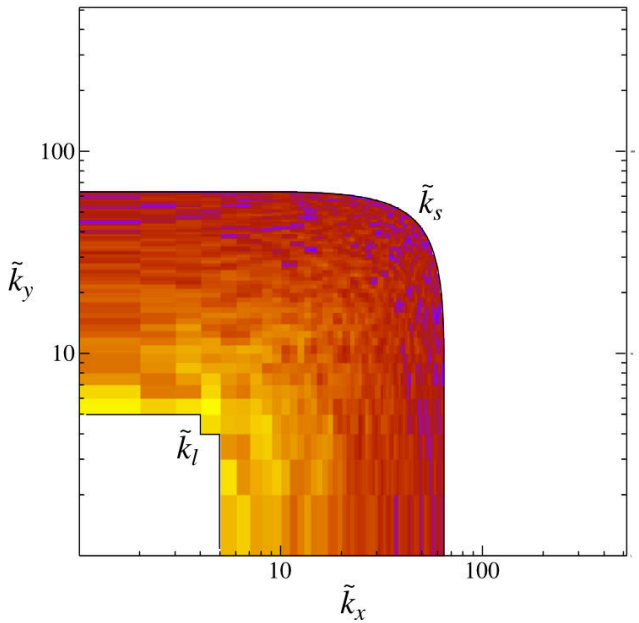


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 32$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



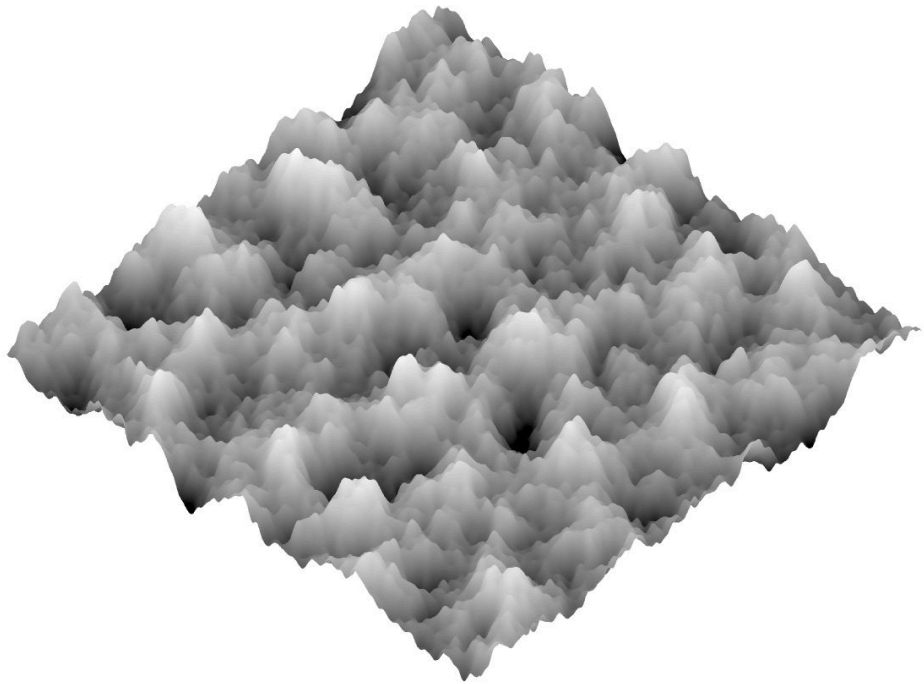
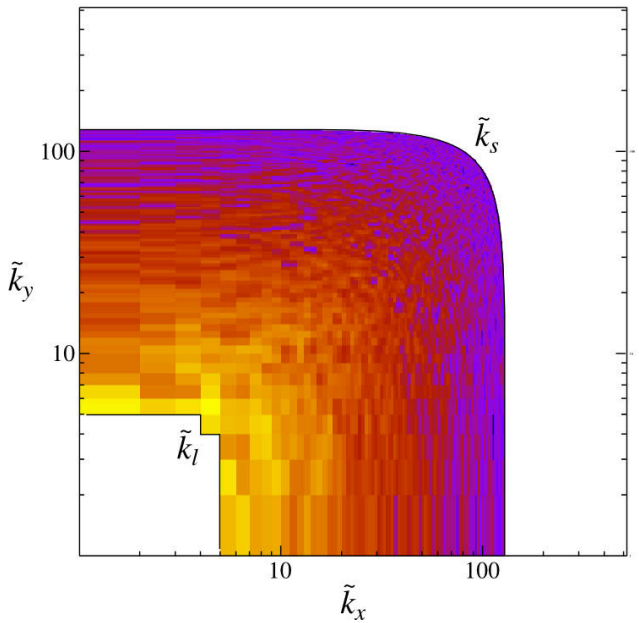


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 64$

Effect of parameters: illustration

- Effect of the high frequency cutoff k_s



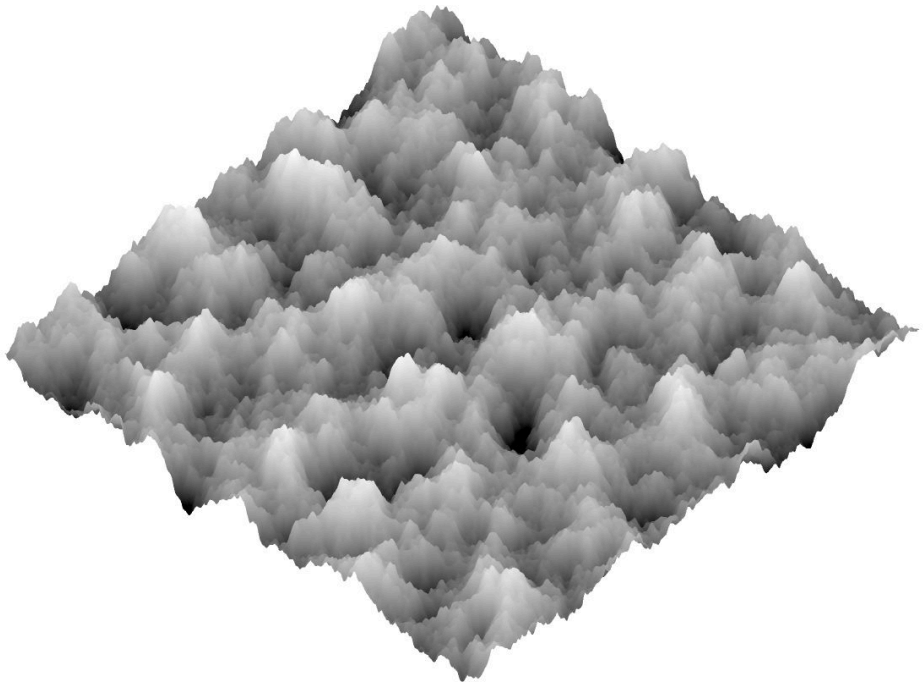


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 4, \quad k_s = 128$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

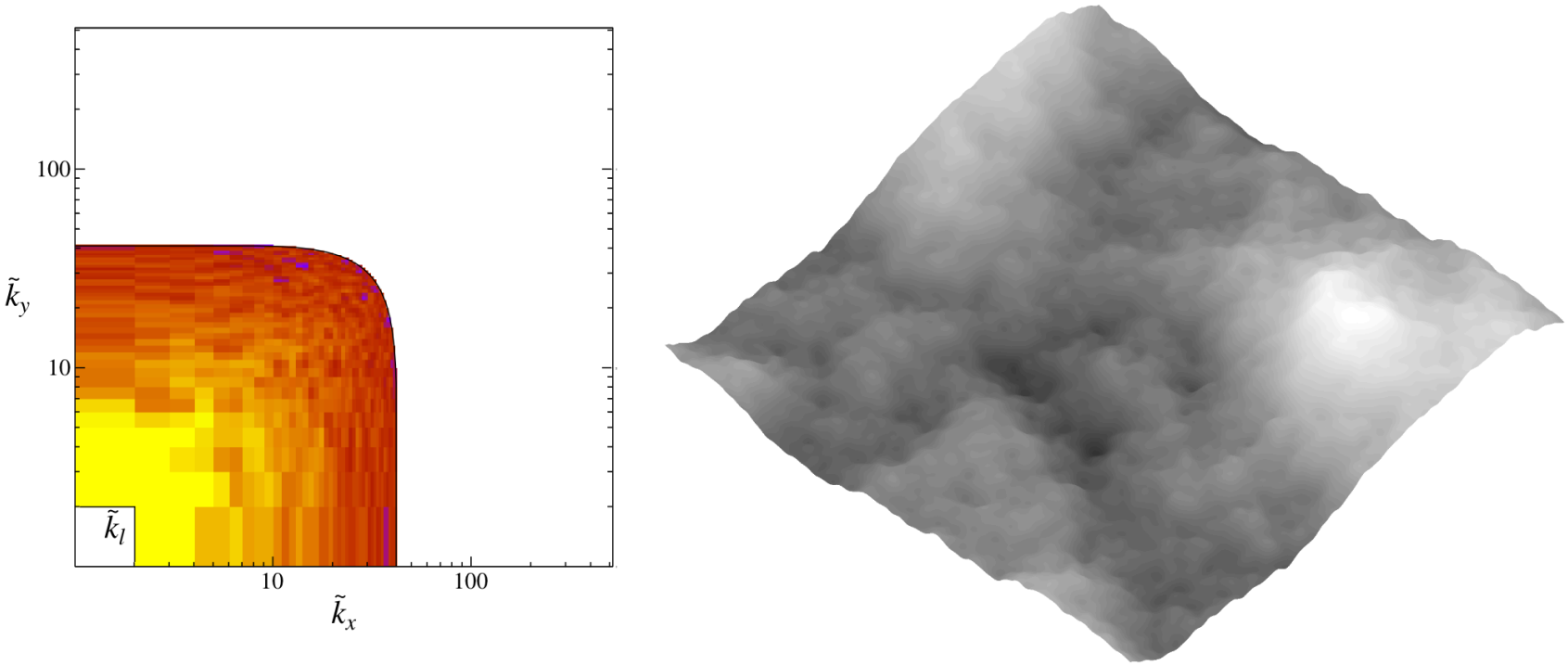
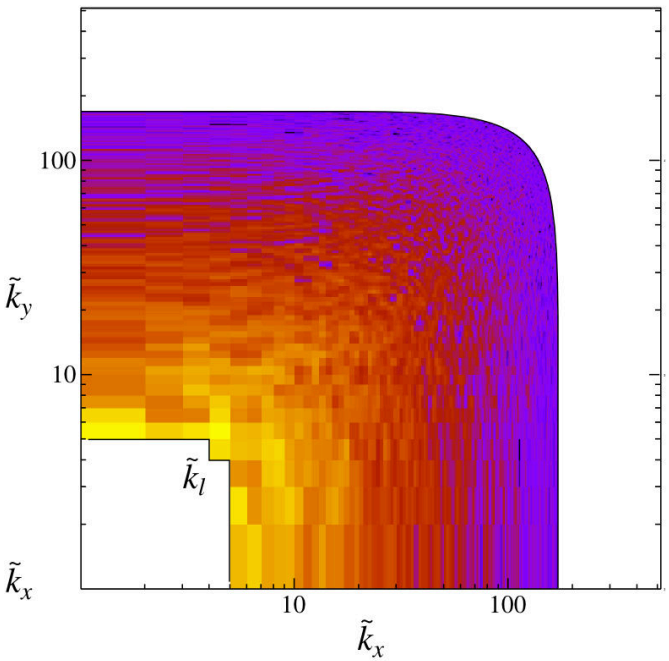


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 1, \quad k_s = 43$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$



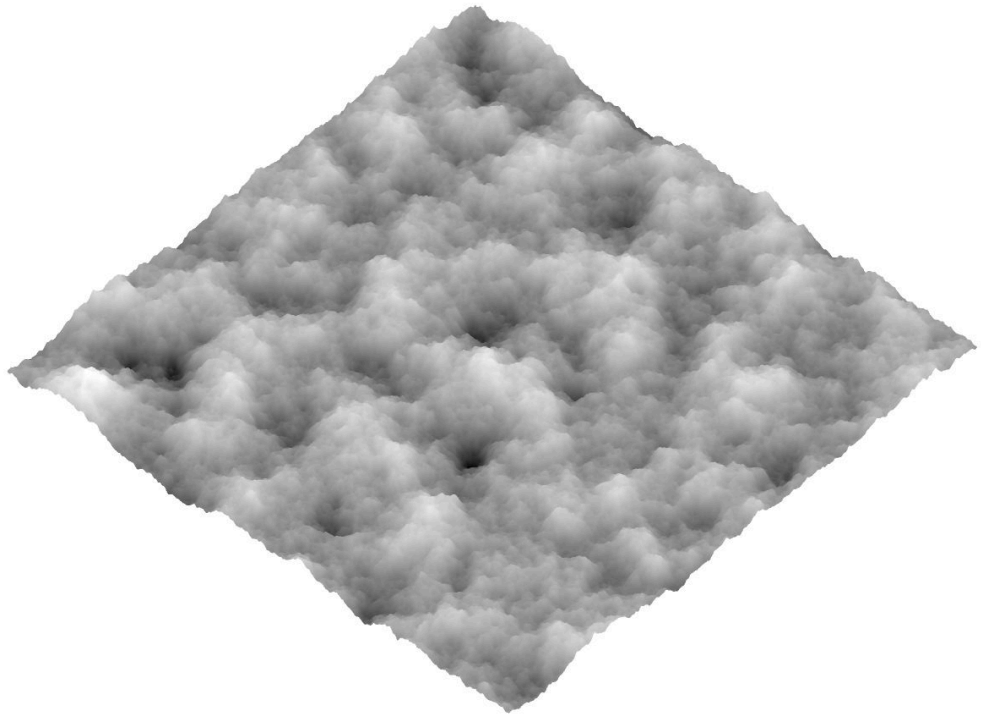


Fig. Power spectral density (Fourier space)
and corresponding rough surface (real space) for
$k_l = 4, \quad k_s = 171$

Effect of parameters: illustration

- Effect of the lower frequency cutoff k_l for $k_s/k_l = \text{const}$

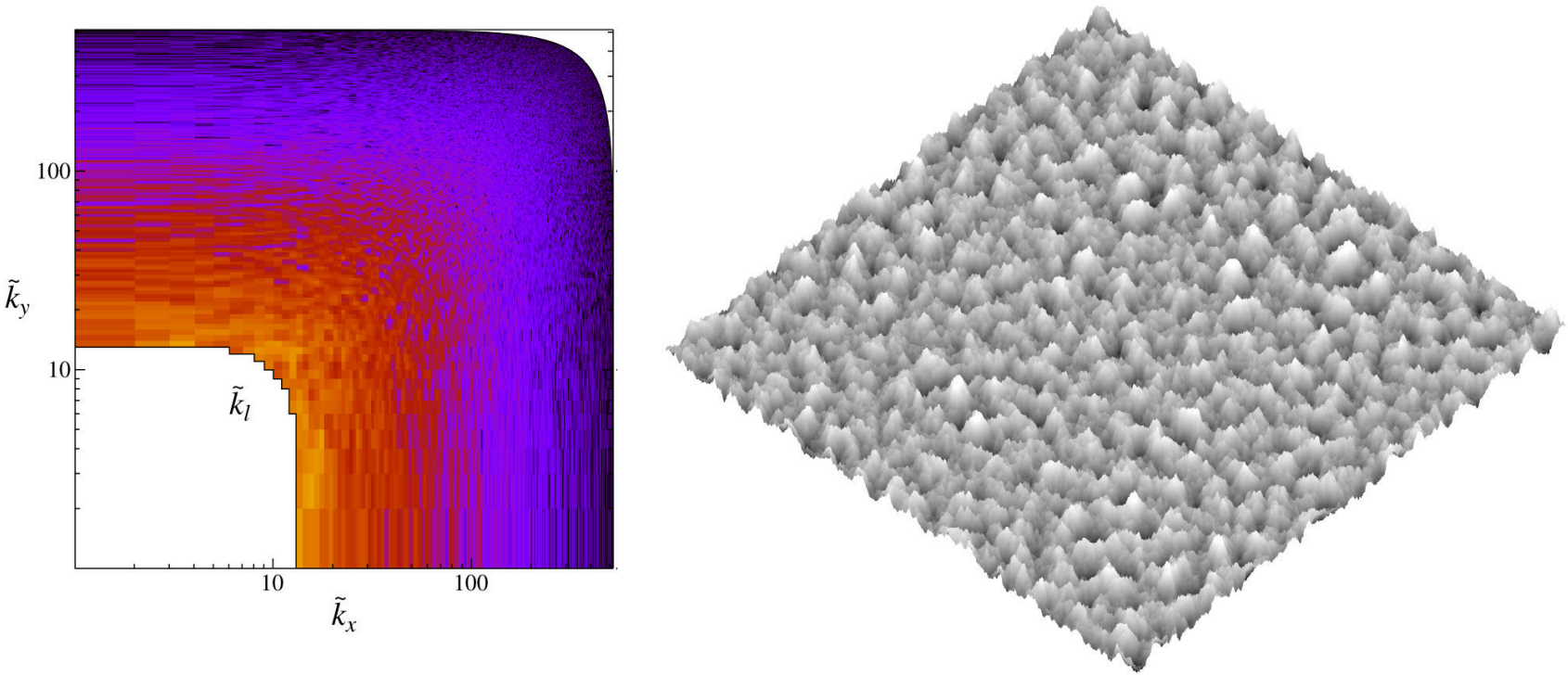


Fig. Power spectral density (Fourier space) and corresponding rough surface (real space) for $k_l = 12, \quad k_s = 512$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

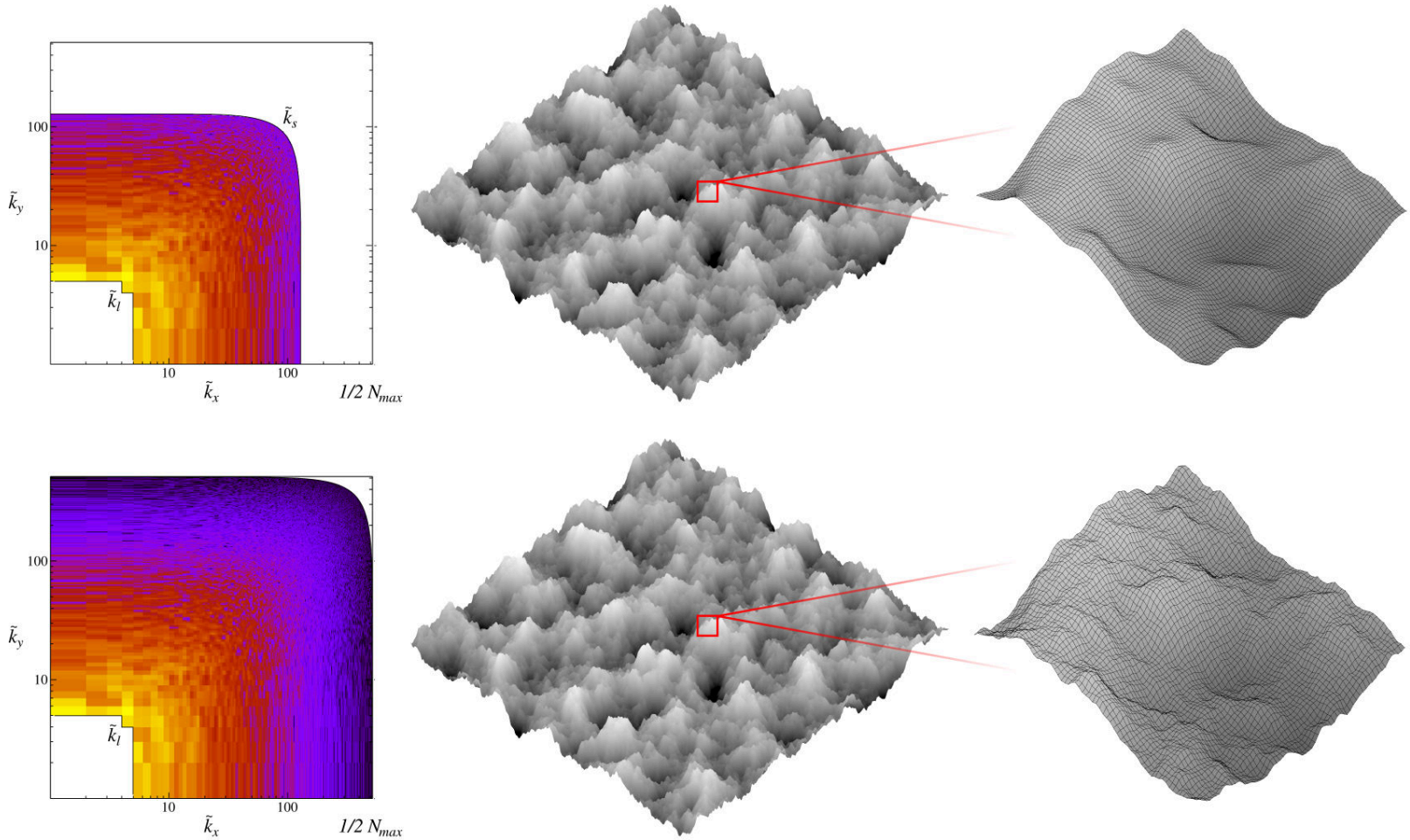


Fig. Power spectral densities (Fourier space) and corresponding rough surfaces (real space) for $k_l = 12, \quad k_s/N = 1/8$ **VS** $k_l = 12, \quad k_s/N = 1/2$

Effect of parameters: illustration

- Effect of the ratio of the higher cutoff to the discretization k_s/N

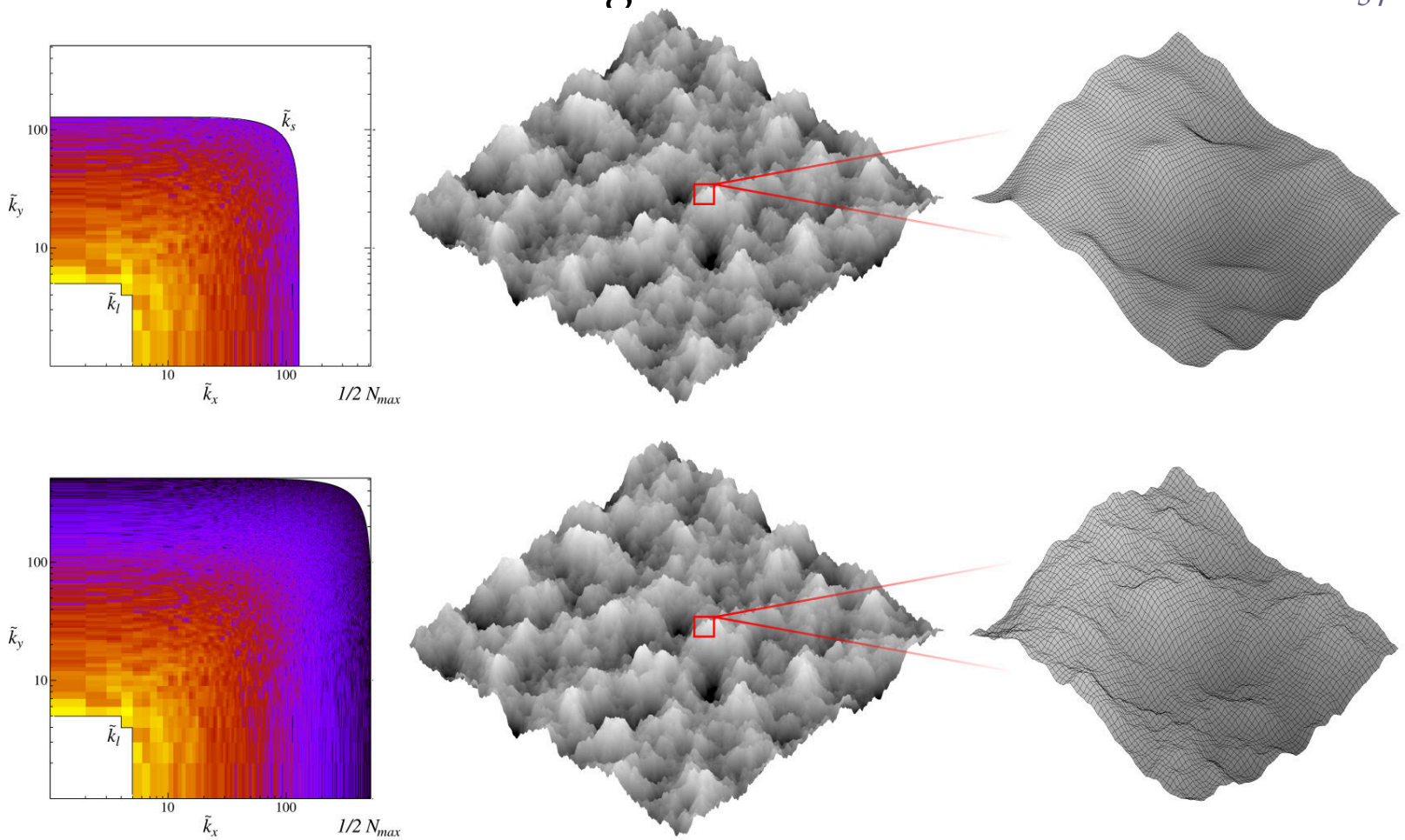


Fig. Power spectral densities (Fourier space)
and corresponding rough surfaces (real space) for
$k_l = 12$, $k_s/N = 1/8$ **(fine)** **VS** $k_l = 12$, $k_s/N = 1/2$ **(too coarse)**
for mechanical simulations

Effect of parameters: illustration

- Effect of the discretisation (single asperity)

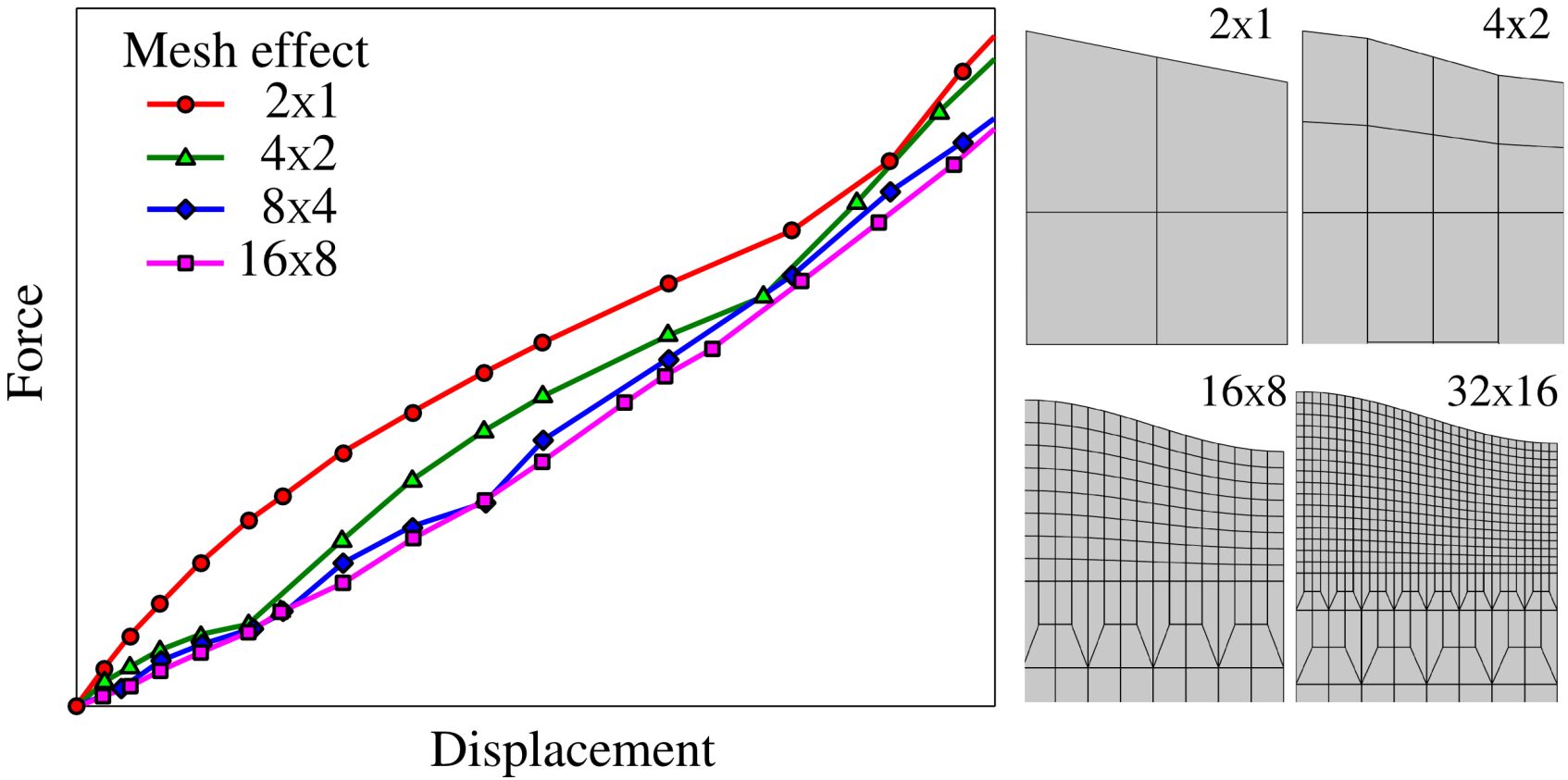


Fig. Effect of the mesh on mechanical response

Effect of parameters: illustration

- Data interpolation (Shanon, bi-cubic Bézier surfaces)

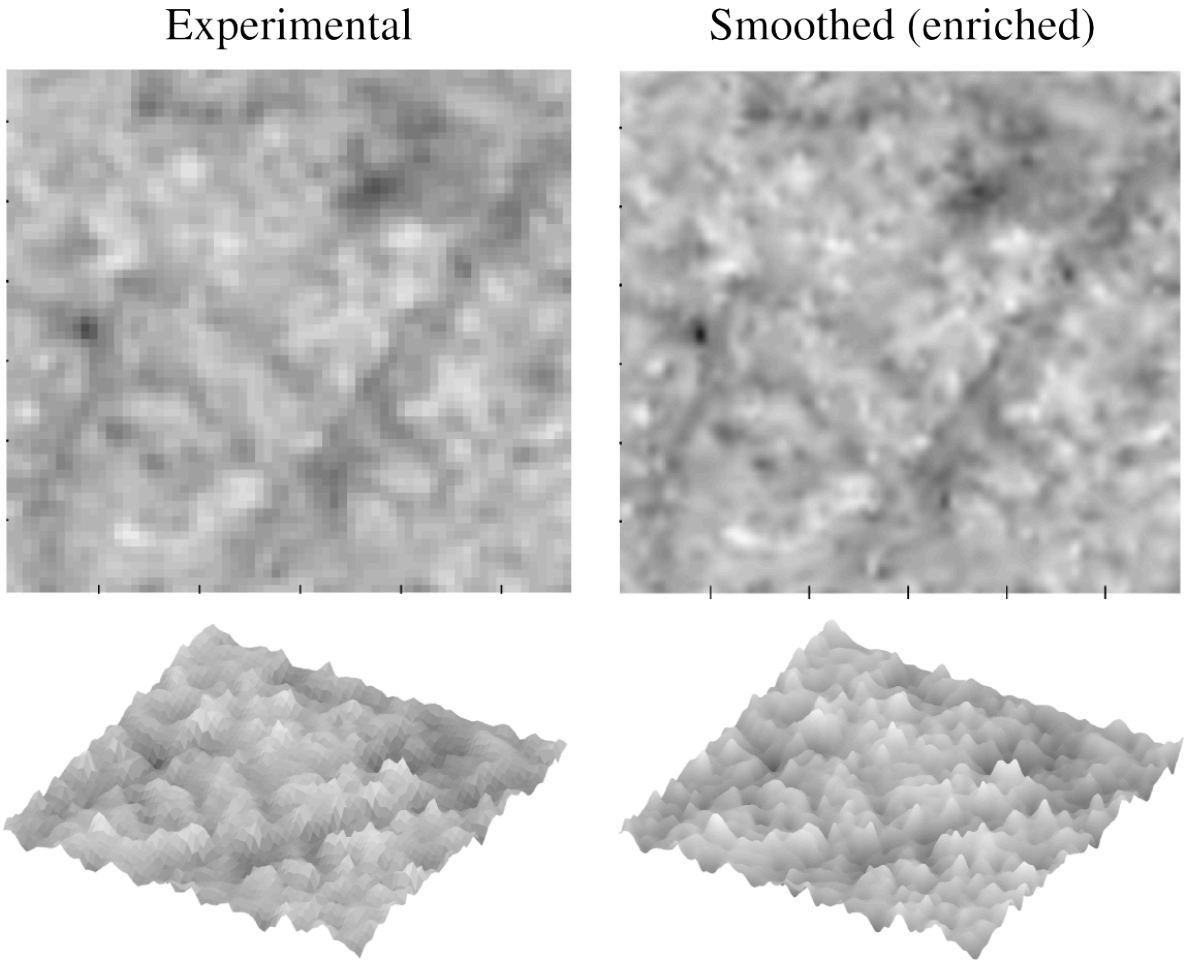


Fig. Bi-cubic Bézier interpolation of an experimental rough surface

[1]Hyun, Robbins, Tribol. Int. (2007)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)

Effect of parameters

Effect of parameters:

- k_l low frequency cutoff
  - representativity/normality[1,2,3]
- k_s high frequency cutoff
  - smoothness and density of asperities
- $\zeta = k_s/k_l$ ratio[3]
  - breadth of the spectrum

$$\alpha \sim \zeta^{2H}$$

Nayak's parameter α is the central characteristic of roughness in asperity based mechanical models.

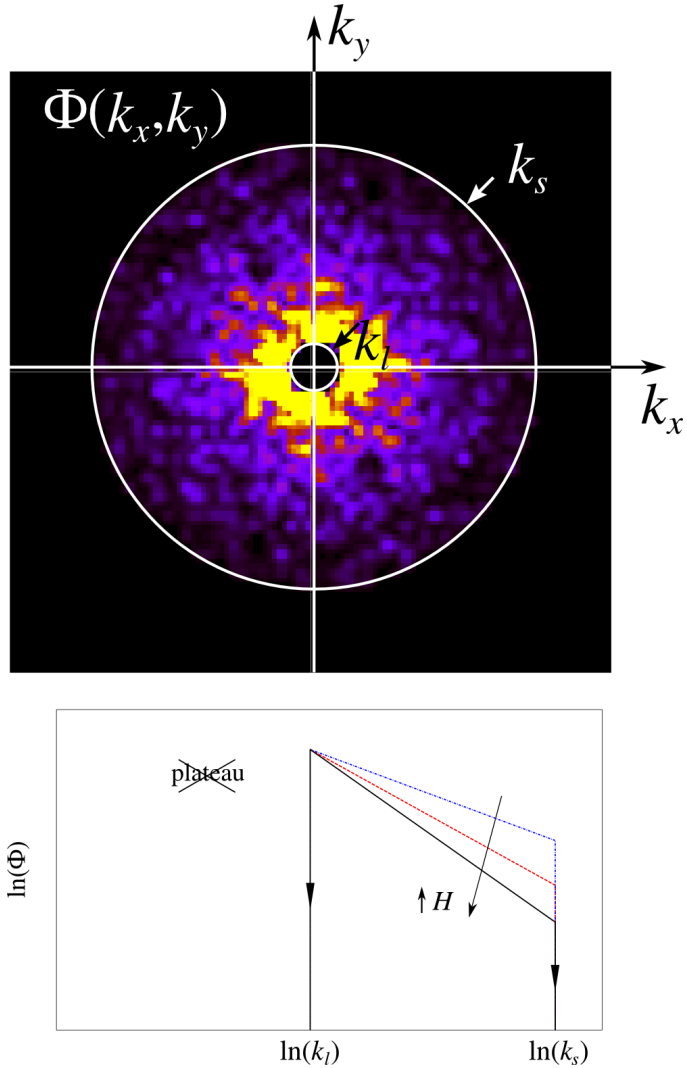


Fig. 3D and radial power spectral densities

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
[3] Yastrebov, Anciaux, Molinari, Phys. Rev. E (2012)
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)

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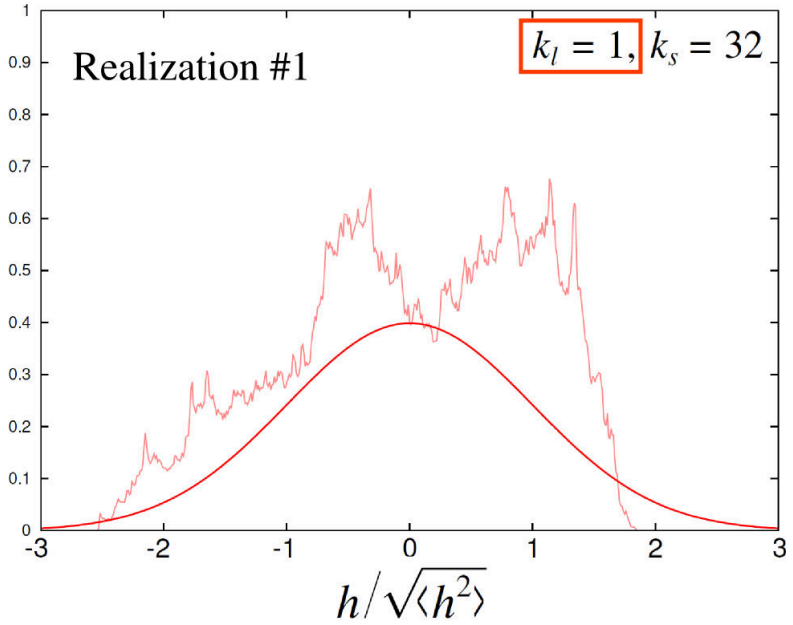

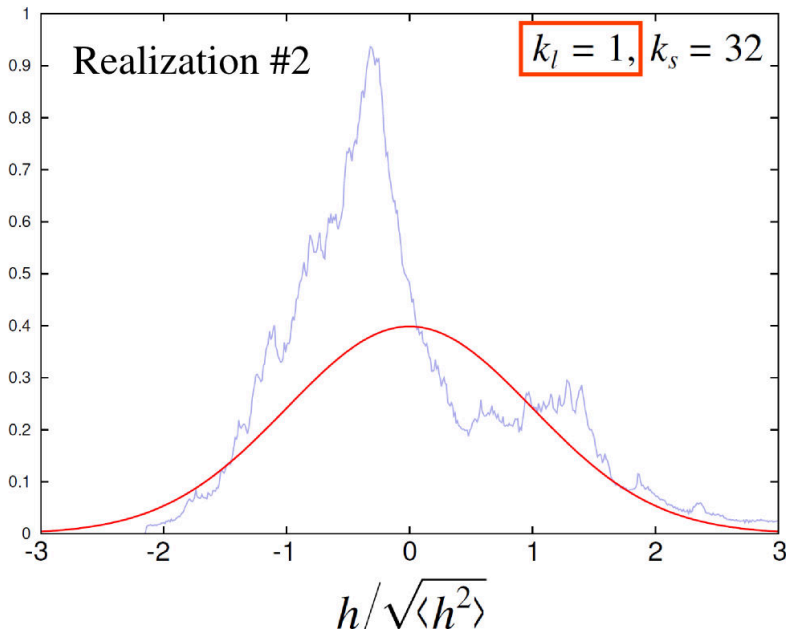

[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
[3] Yastrebov, Anciaux, Molinari, Phys. Rev. E (2012)
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)

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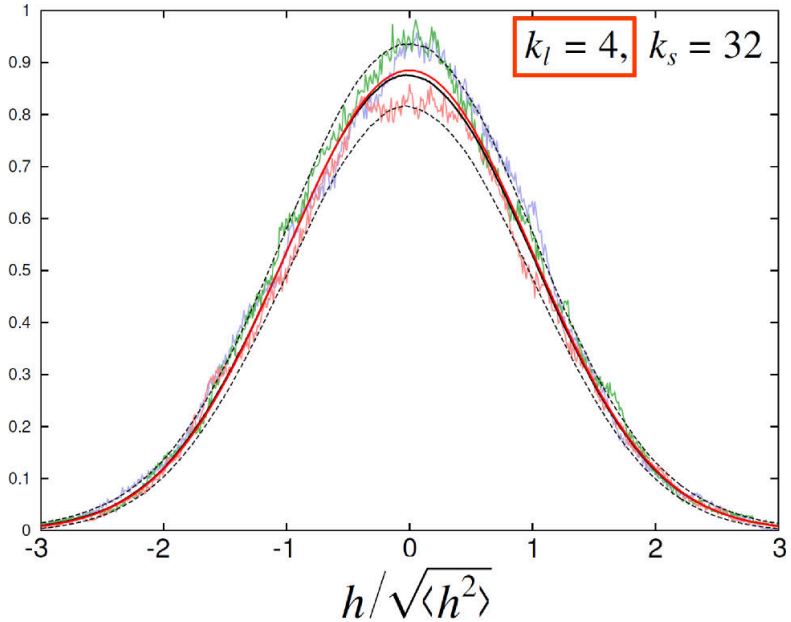
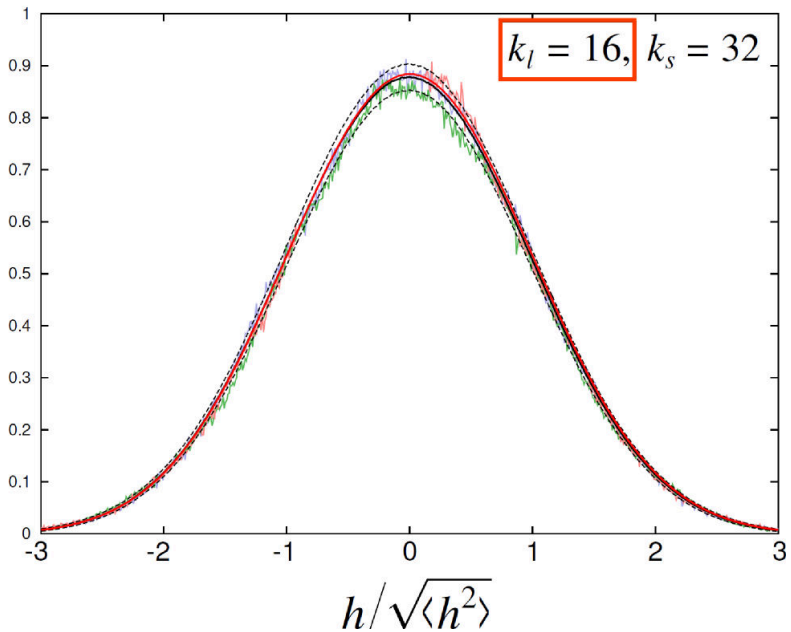




[1] Vallet, Lasseux, Sainsot, Zahouani, Tribol. Int. (2009)
[2] Yastrebov, Durand, Proudhon, Cailletaud, C.R. Mécan. (2011)
[3] Yastrebov, Anciaux, Molinari, Phys. Rev. E (2012)
[4] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. (2015)

Interconnection of parameters

- Spectral moment and k_l, k_s, H:

$$m_{0p} \approx m_{p0} \approx \Phi_0 \int_{k_l}^{k_s} \int_{0}^{2\pi} [k\cos(\varphi)]^p \, (k/k_l)^{-2(1+H)} \, k dk d\varphi = \Phi_0 k_l^{p+2} \frac{\zeta^{p-2H} - 1}{p - 2H} T(p)$$

$$\text{with} \quad T(p) = \int_0^{2\pi} \cos^p(\varphi) d\varphi = \begin{cases} 2\pi, & \text{if } p = 0; \\ \pi, & \text{if } p = 2; \\ 3\pi/4, & \text{if } p = 4. \end{cases}$$

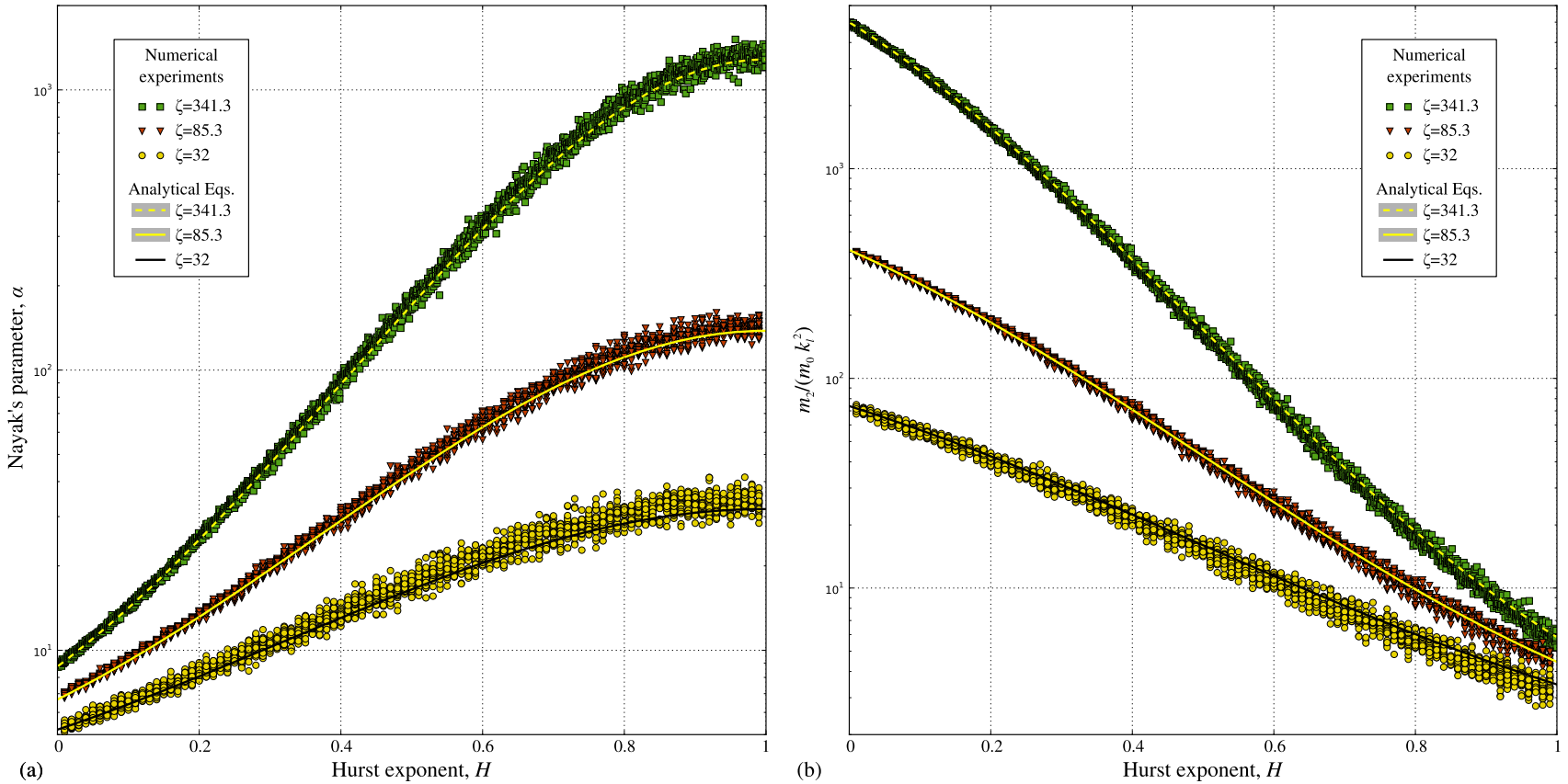
- Nayak's parameter

$$\alpha(H, \zeta) = \frac{3}{2} \frac{(1-H)^2}{H(H-2)} \frac{(\zeta^{-2H} - 1)(\zeta^{4-2H} - 1)}{(\zeta^{2-2H} - 1)^2}$$

- Asperity density

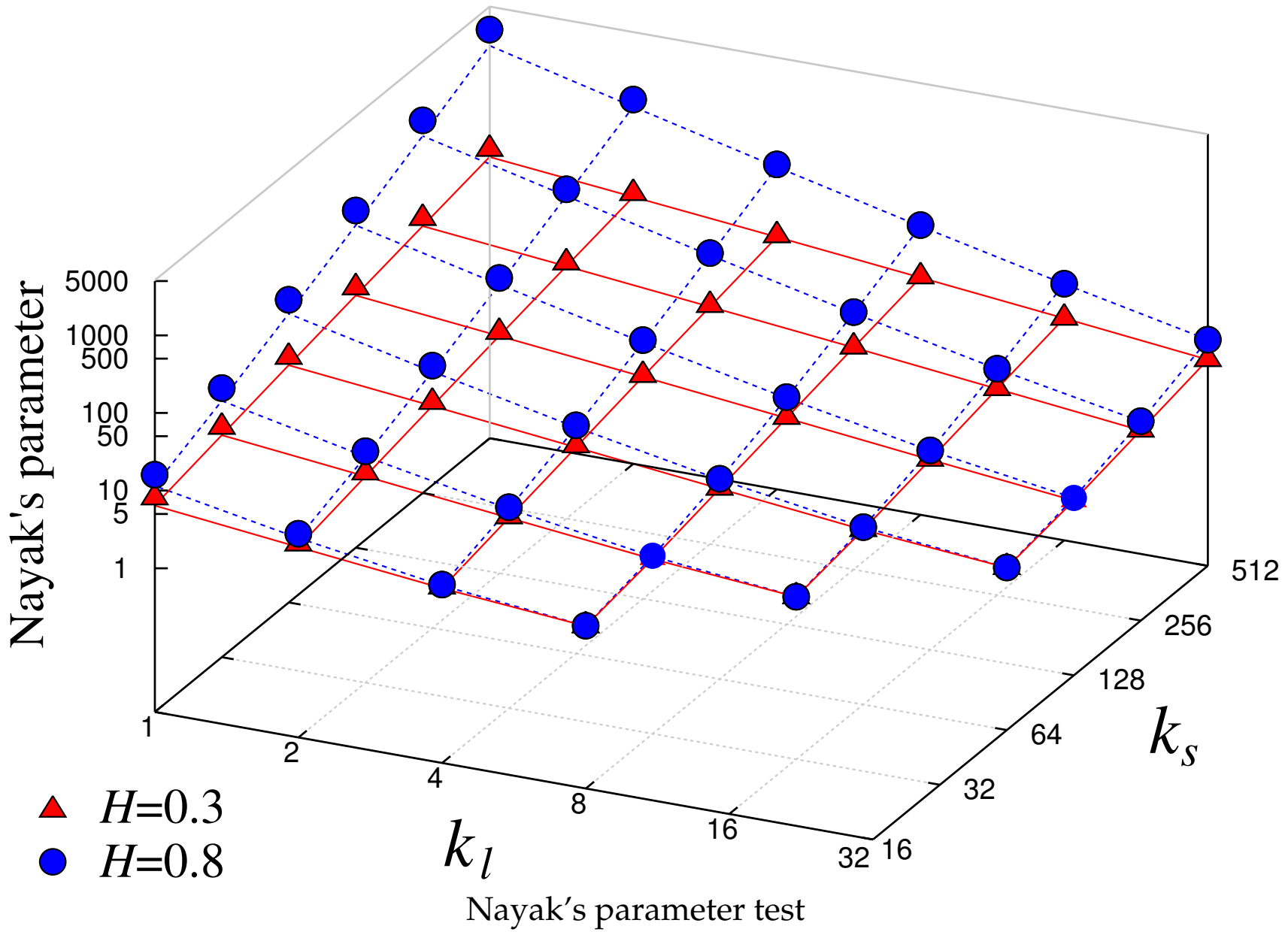
$$D = \frac{\sqrt{3}}{18\pi} \frac{m_4}{m_2} = \frac{\sqrt{3}}{24\pi} \frac{1-H}{2-H} \frac{\zeta^{4-2H} - 1}{\zeta^{2-2H} - 1} k_l^2$$

Interconnection of parameters

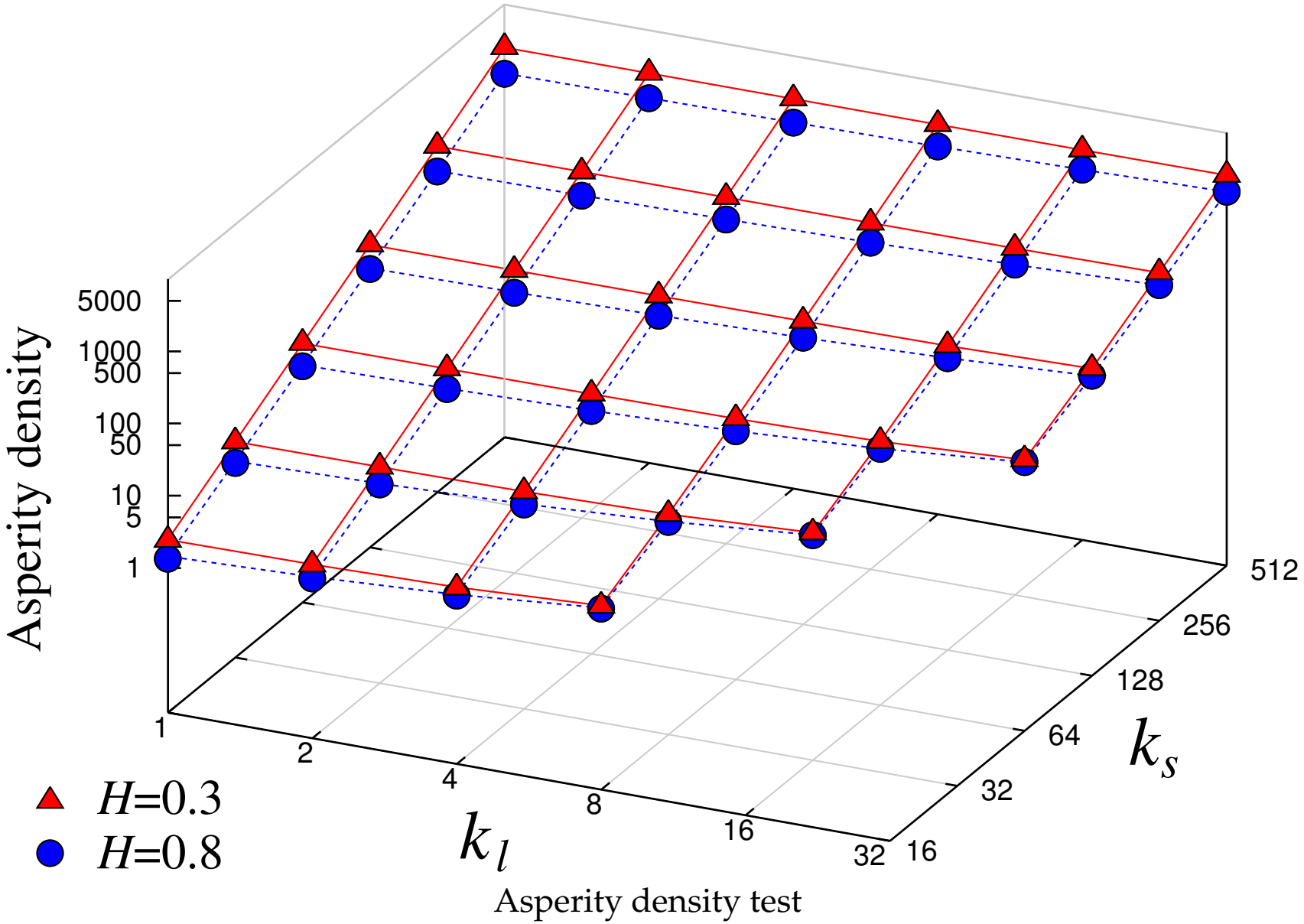


Numerical verification on 100 000 generated rough surfaces with 2048×2048 points

Interconnection of parameters



Interconnection of parameters



Asperity analysis

- Detect summits (z_{ij} higher than neighbouring points) and evaluate second derivatives

$$\frac{\partial^2 z}{\partial x^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}; \quad \frac{\partial^2 z}{\partial y^2} = \frac{z_{i+1j} + z_{i-1j} - 2z_{ij}}{2\Delta x^2}$$

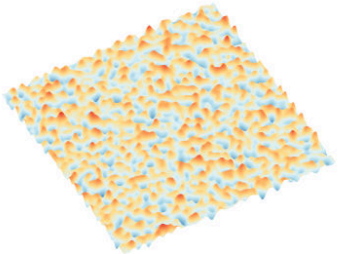
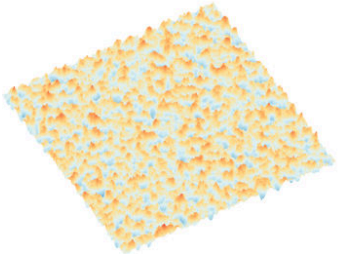
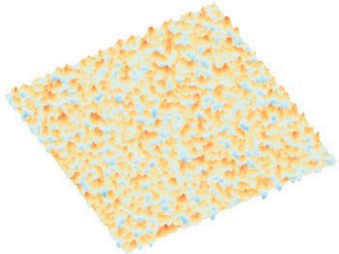
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{z_{i+1j+1} + z_{i+1j+1} - z_{i+1j-1} - z_{i-1j+1}}{4\Delta x^2}$$

- Principal curvatures $\kappa_{1,2}$:

$$\kappa_{1,2} = \frac{1}{2}\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) \pm \sqrt{\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + \frac{1}{4}\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}\right)^2}$$

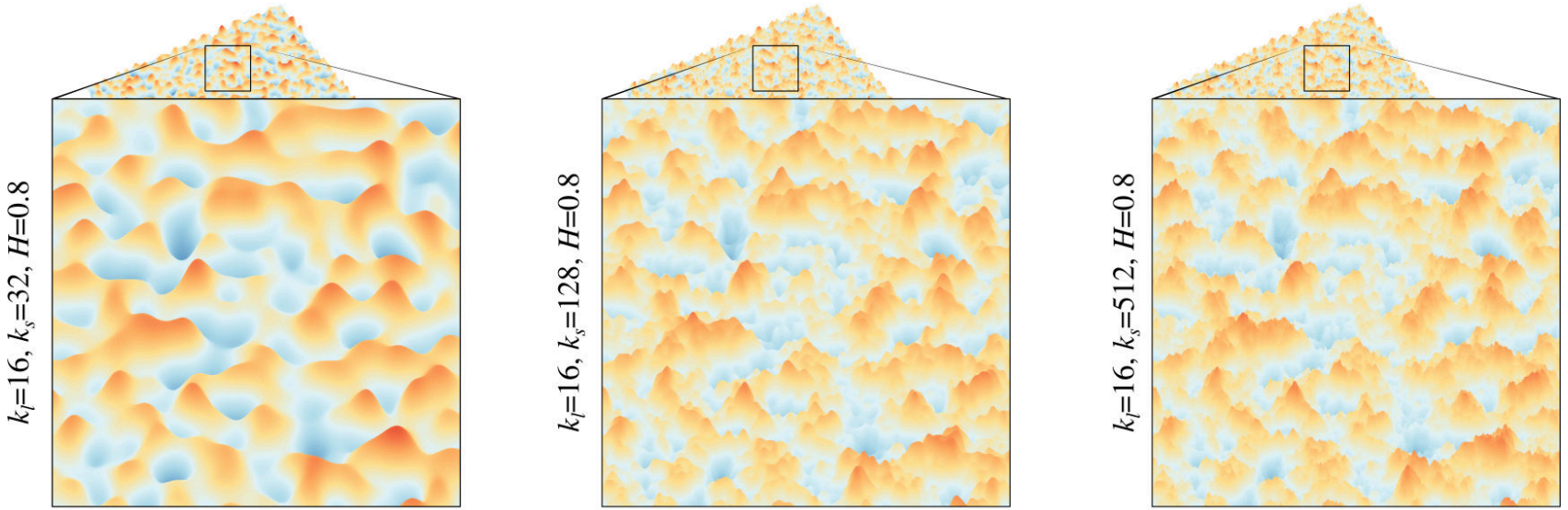
- Saddle point $\kappa_1 \kappa_2 < 0$, extrema $\kappa_1 \kappa_2 > 0$
- Mean curvature which can be safely used in Hertz theory: $\bar{\kappa} = \sqrt{\kappa_1 \kappa_2}$

Asperity analysis

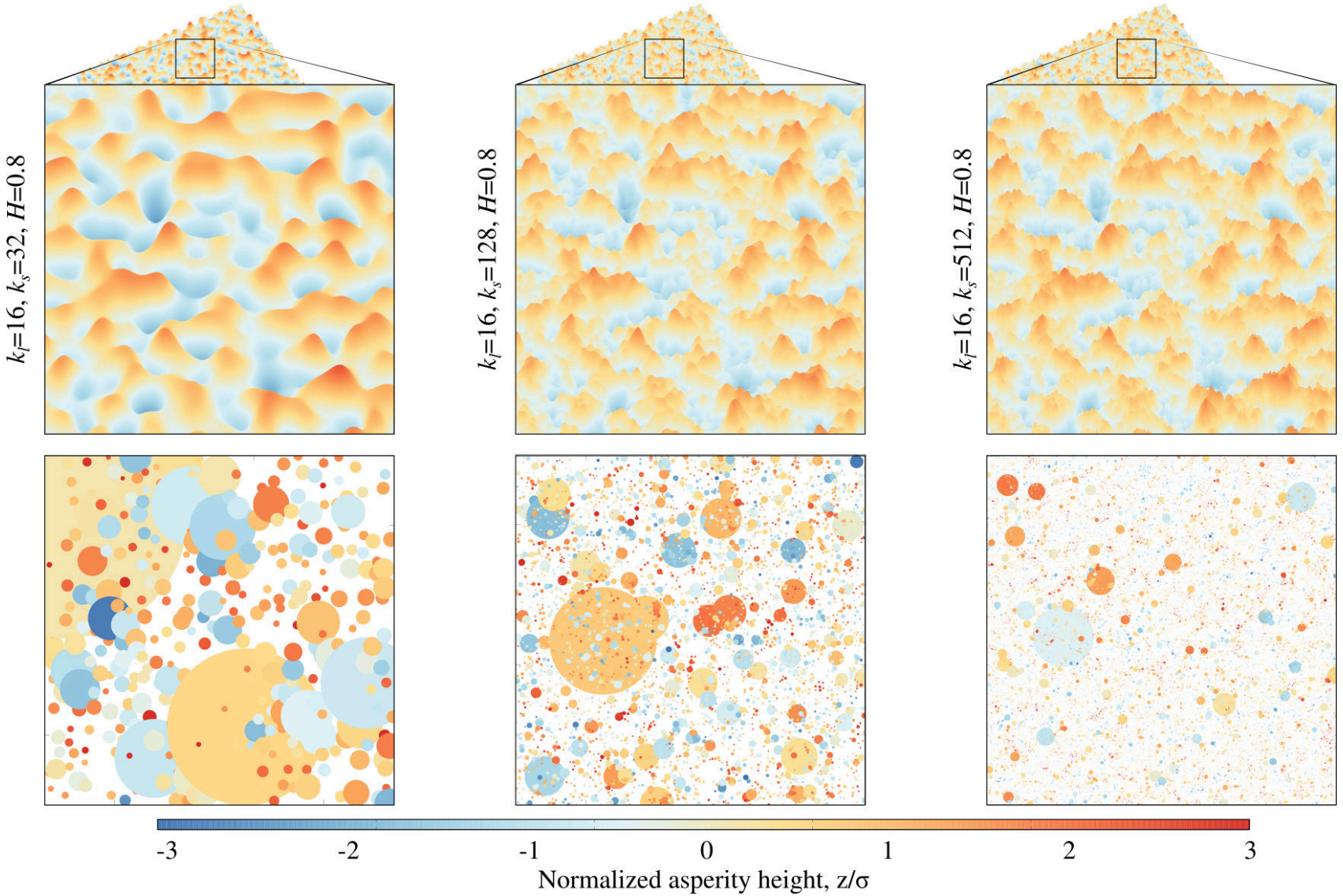
Rough surfaces and associated asperities

Asperity analysis



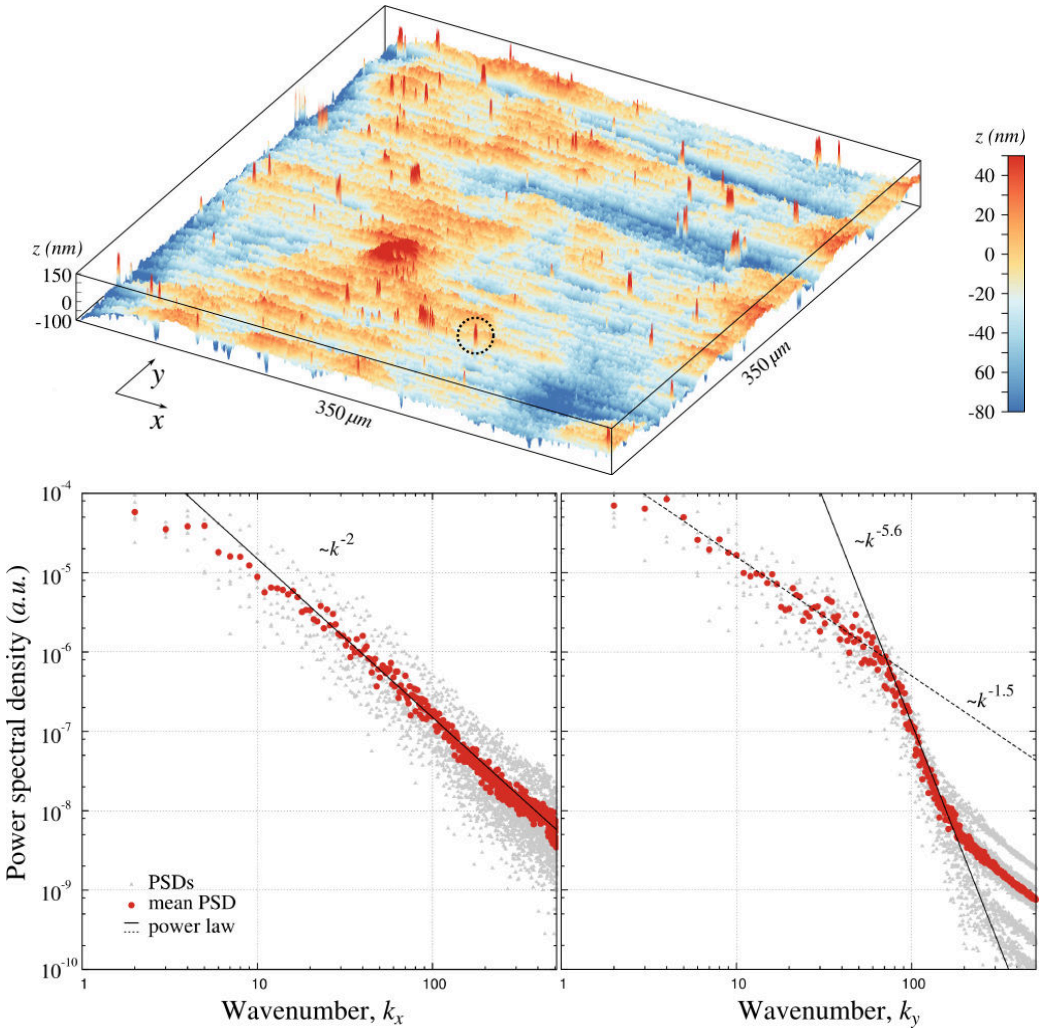
Rough surfaces and associated asperities

Asperity analysis



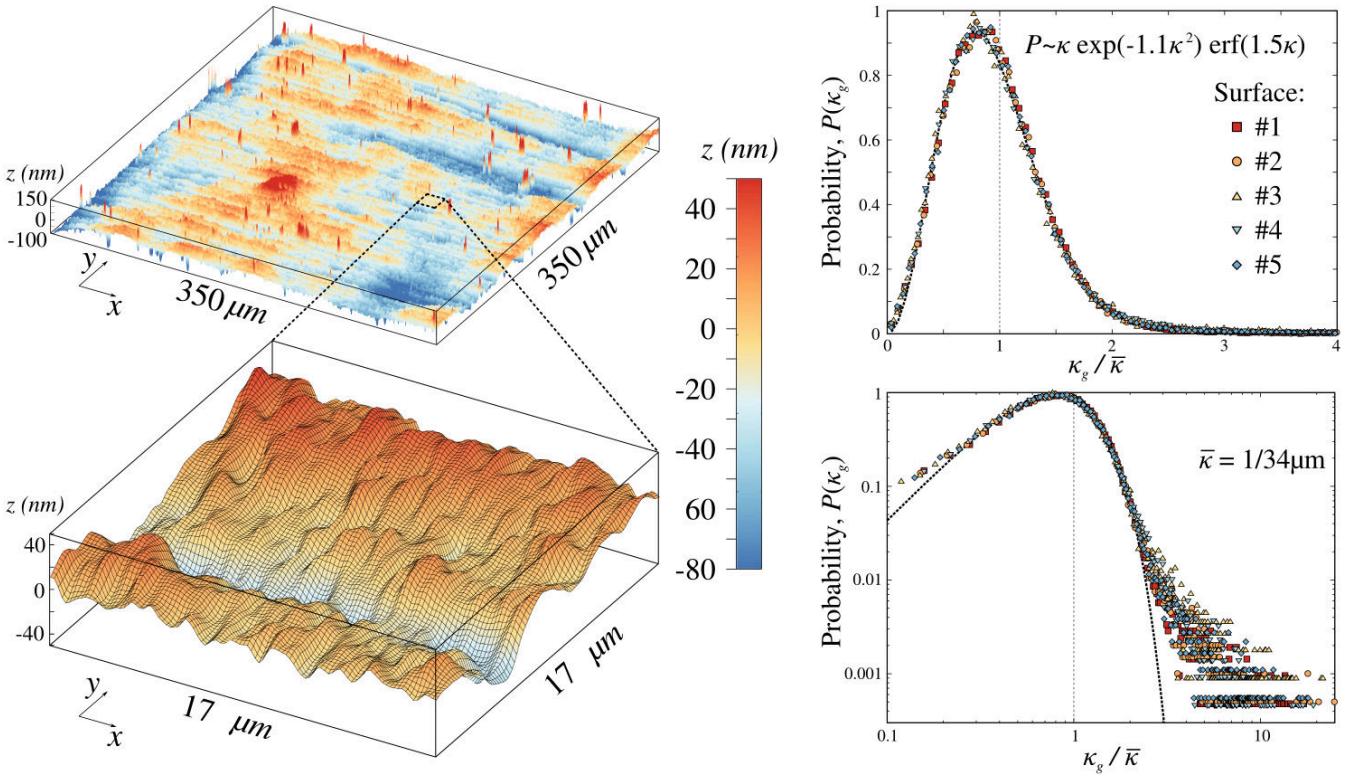
Rough surfaces and associated asperities

Examples



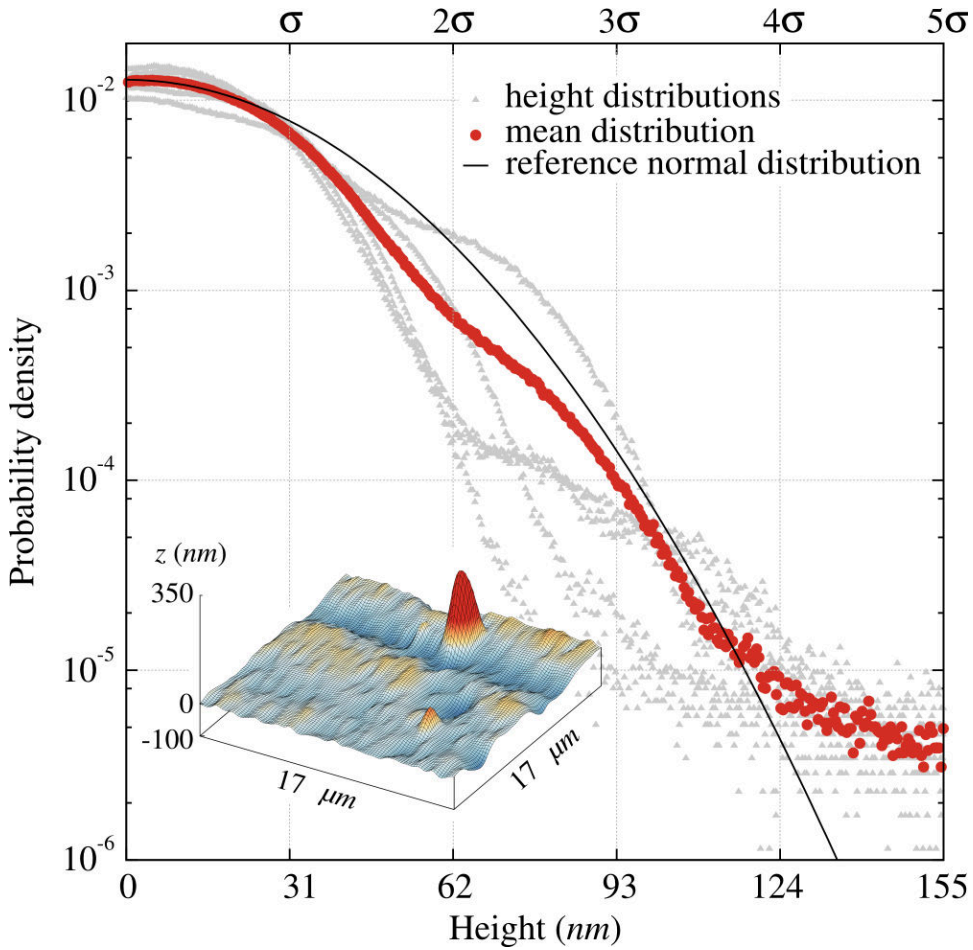
[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

Examples



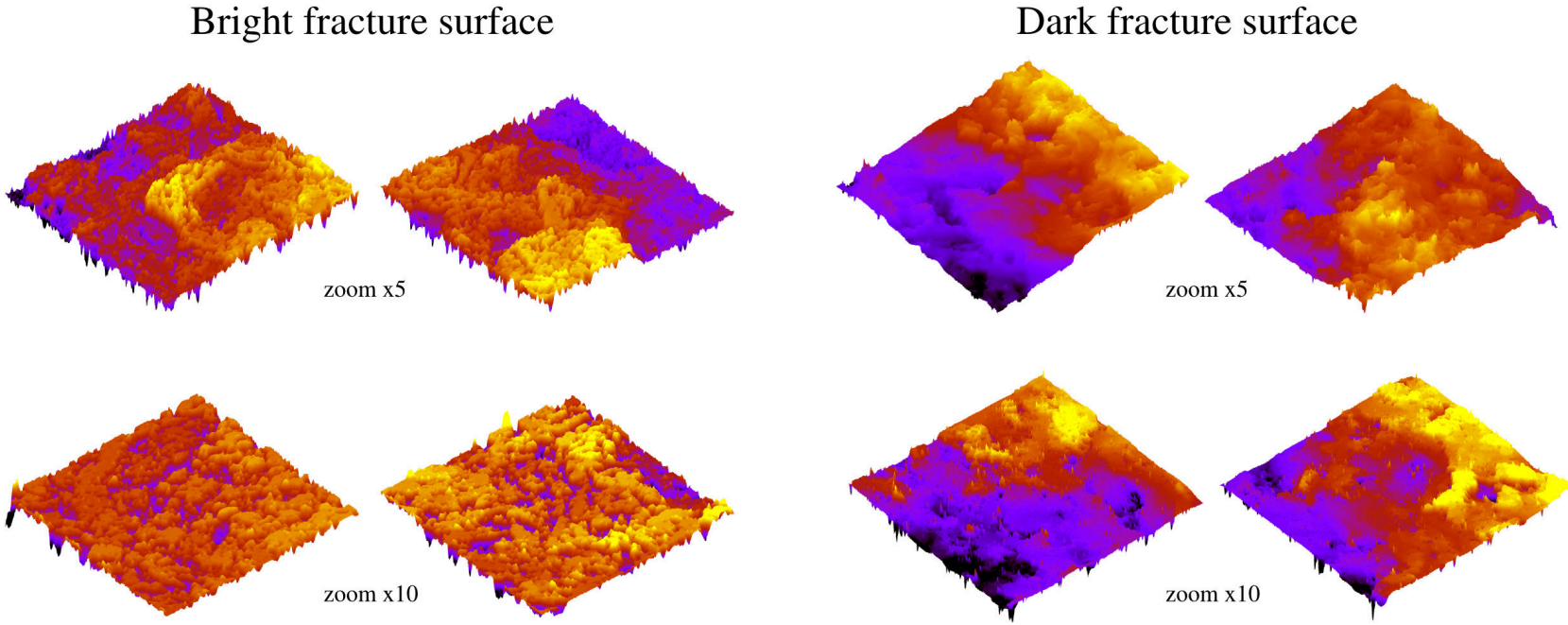
Asperity curvatures

Examples



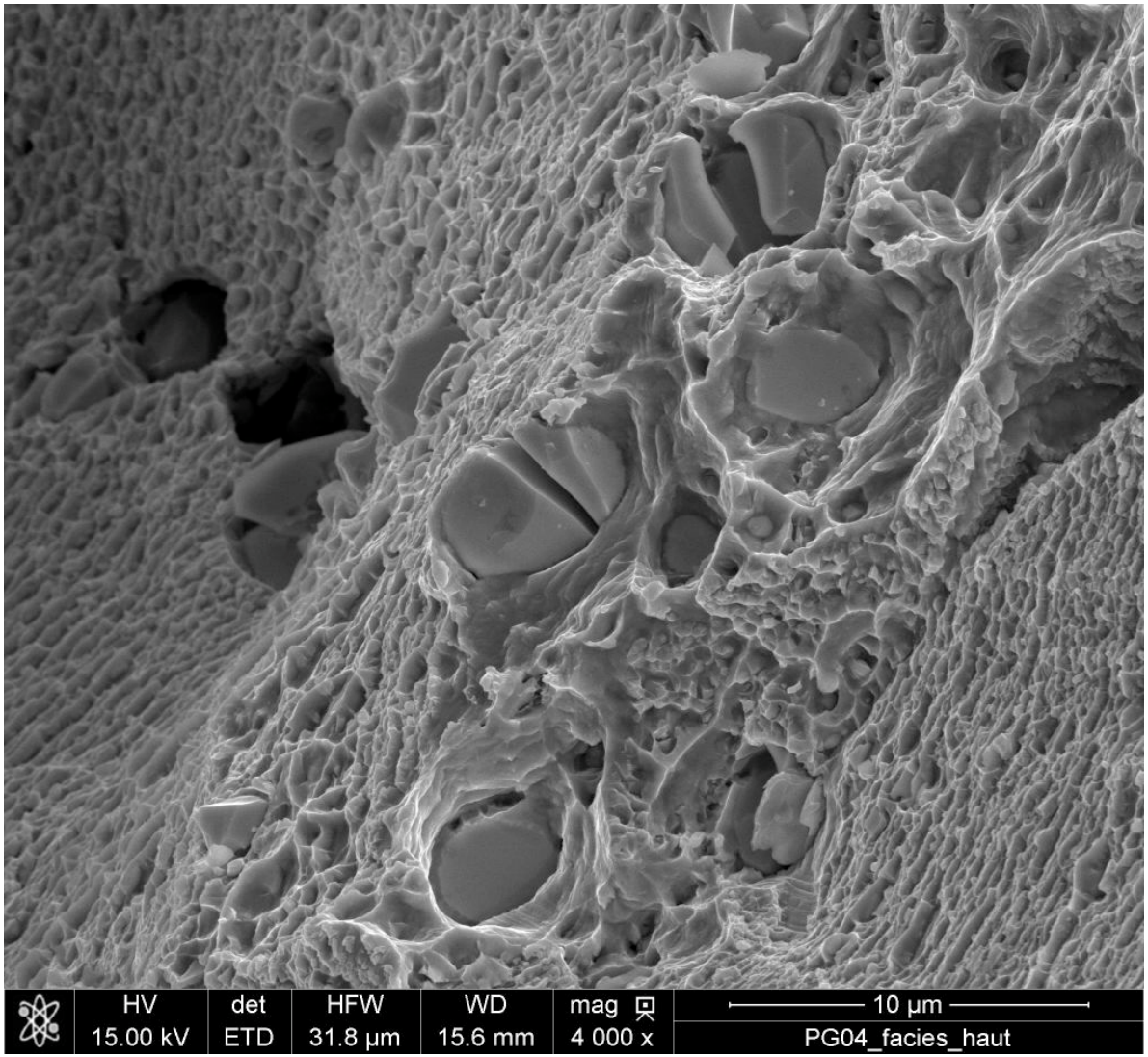
[1] Yastrebov et al, Three-level multi-scale modeling of electrical contacts sensitivity study and experimental validation, Proceedings of Holm Conference, 2015.

Examples



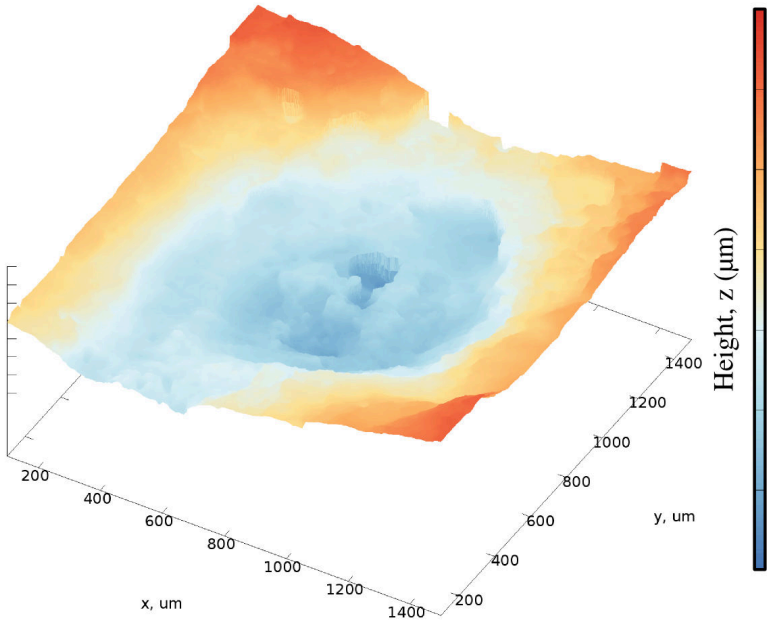
Fatigue & creep fracture surfaces (Ti-alloy)
in collaboration with A. Marchenko

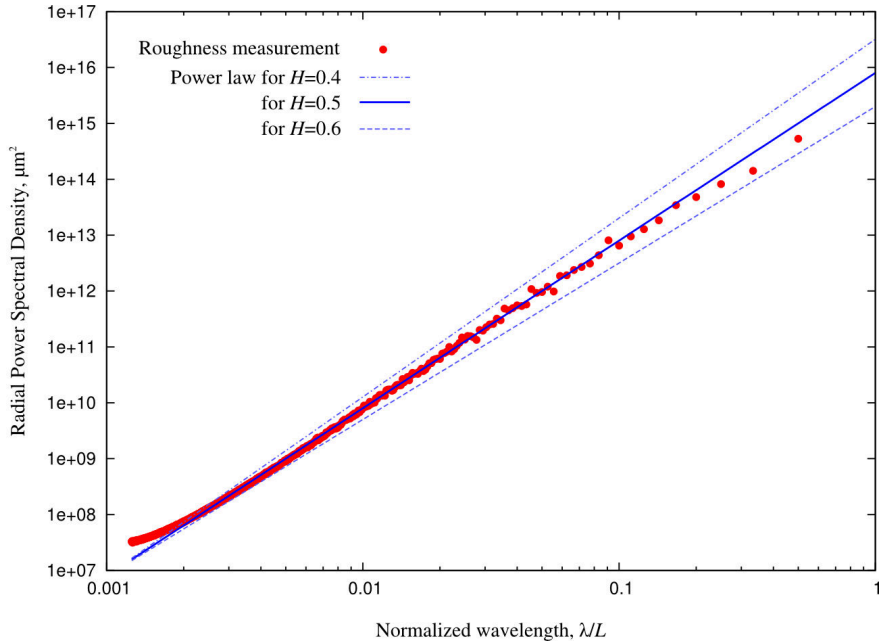
Examples



Fatigue fracture surface (Co-alloy), particles WC
Courtesy of V. Esin

Examples



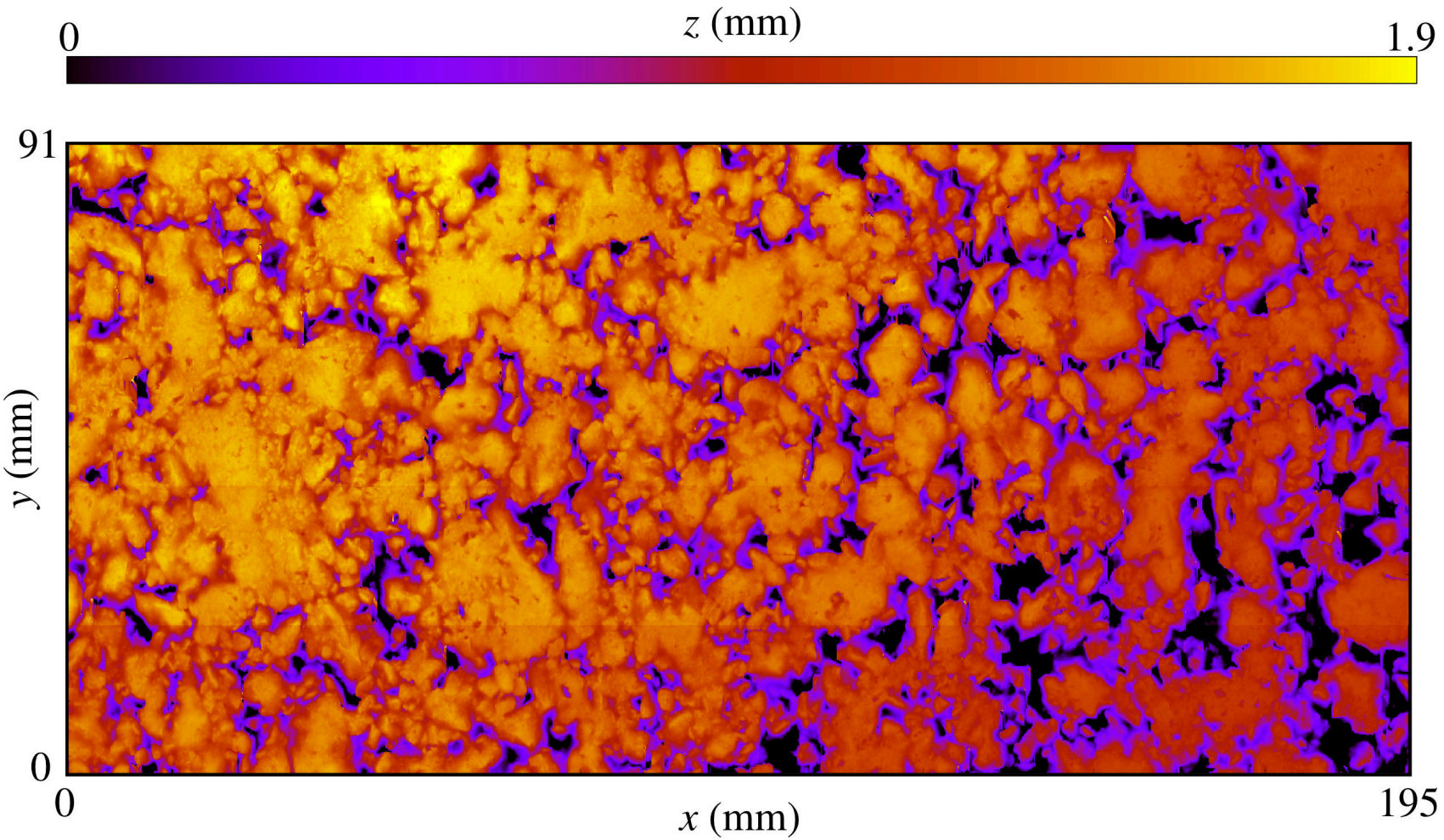


Crater topography and PSD

In collaboration with D. Tkalich (NTNU, Sintef)

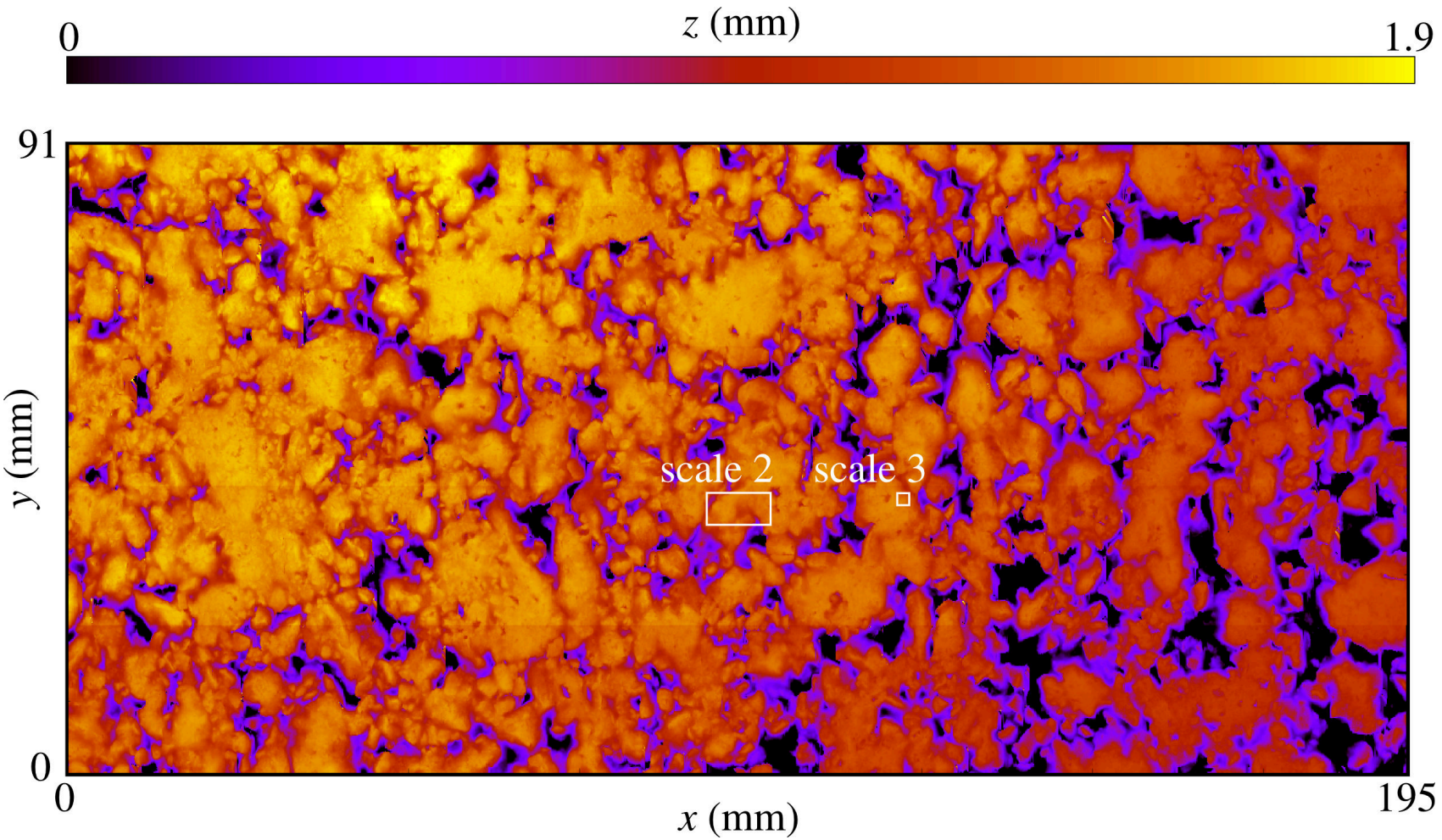
Multiscale road roughness

- Scale 1 (x1)



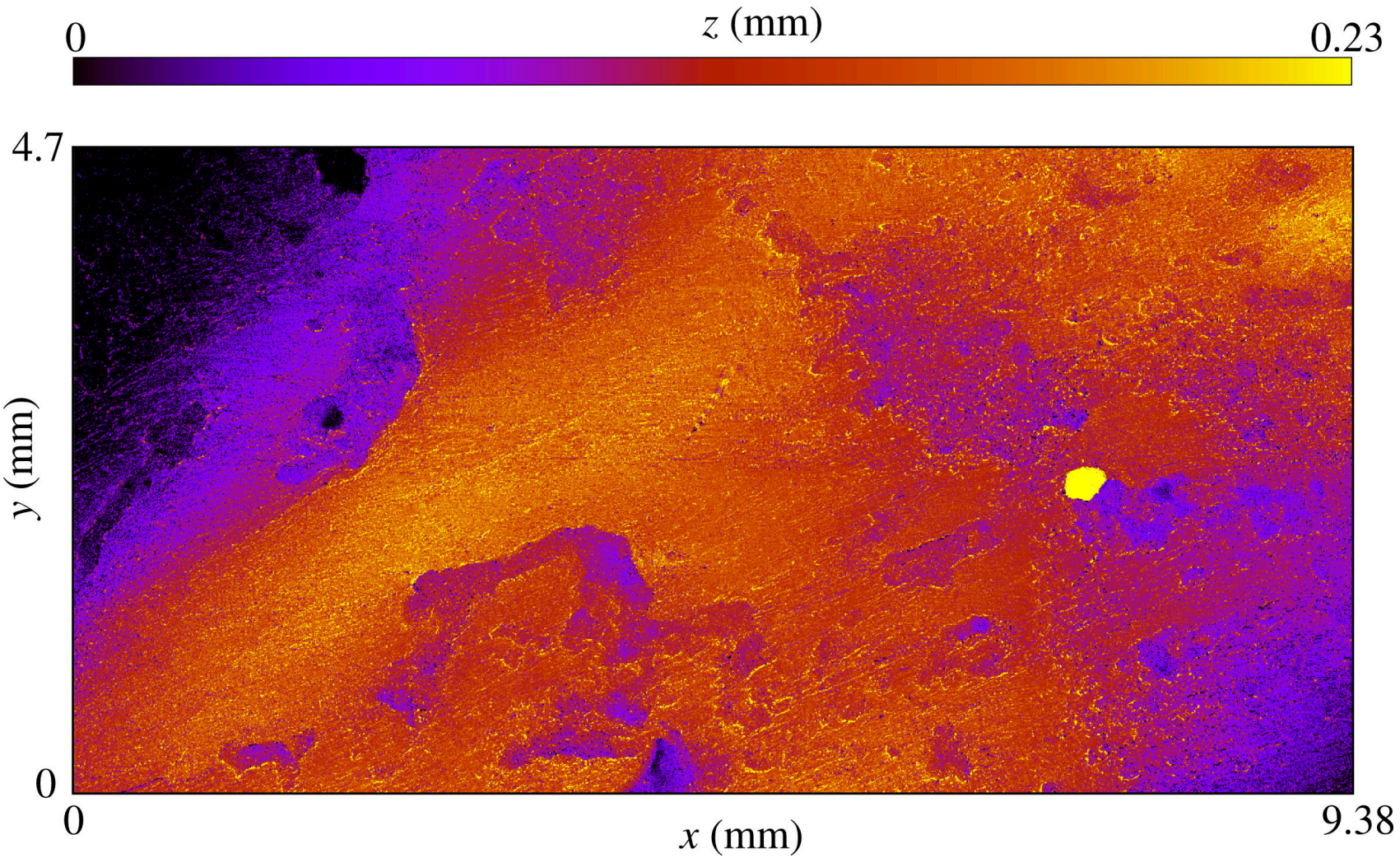
Multiscale road roughness

- Scale 1 (x1)



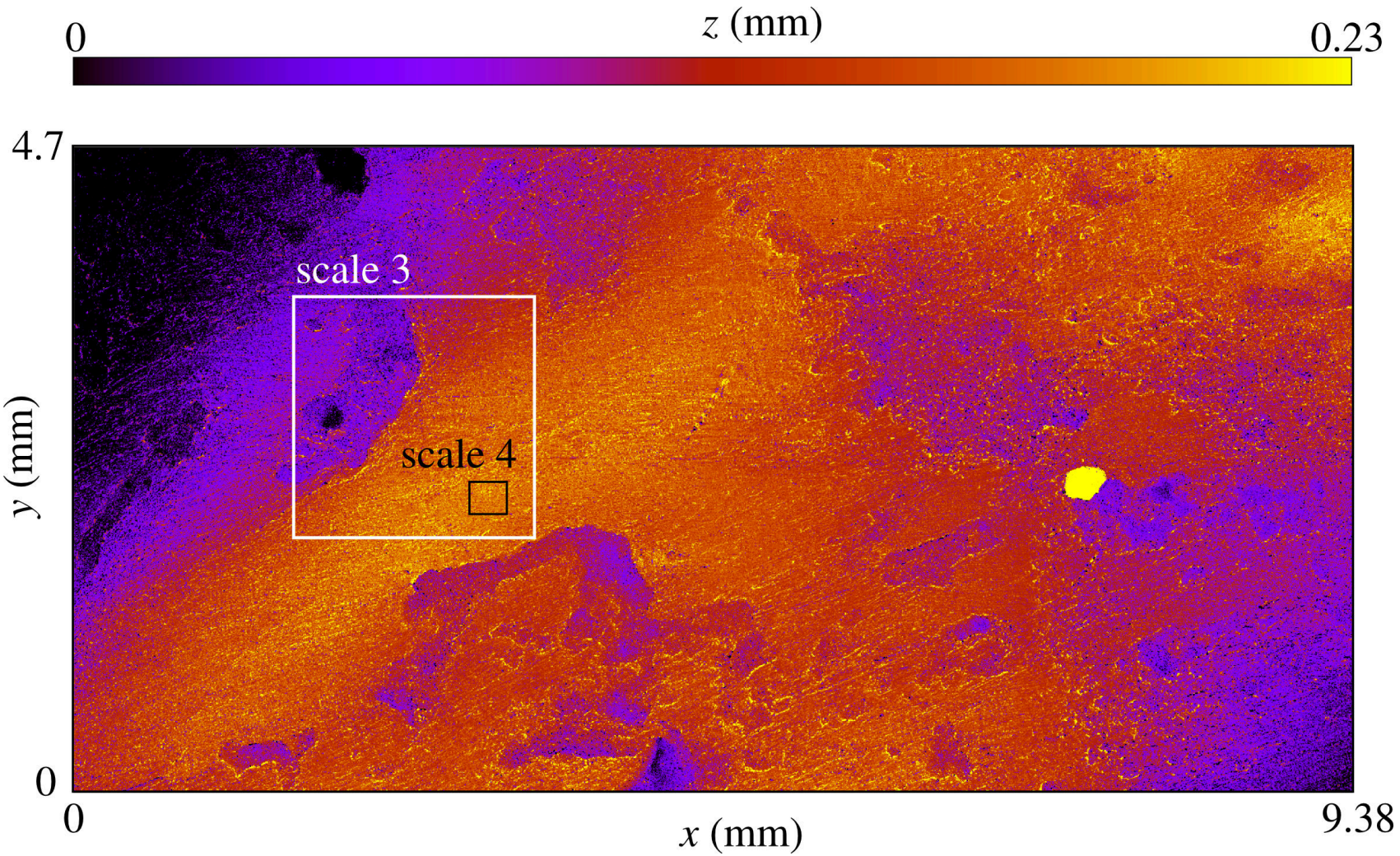
Multiscale road roughness

- Scale 2 (x5)



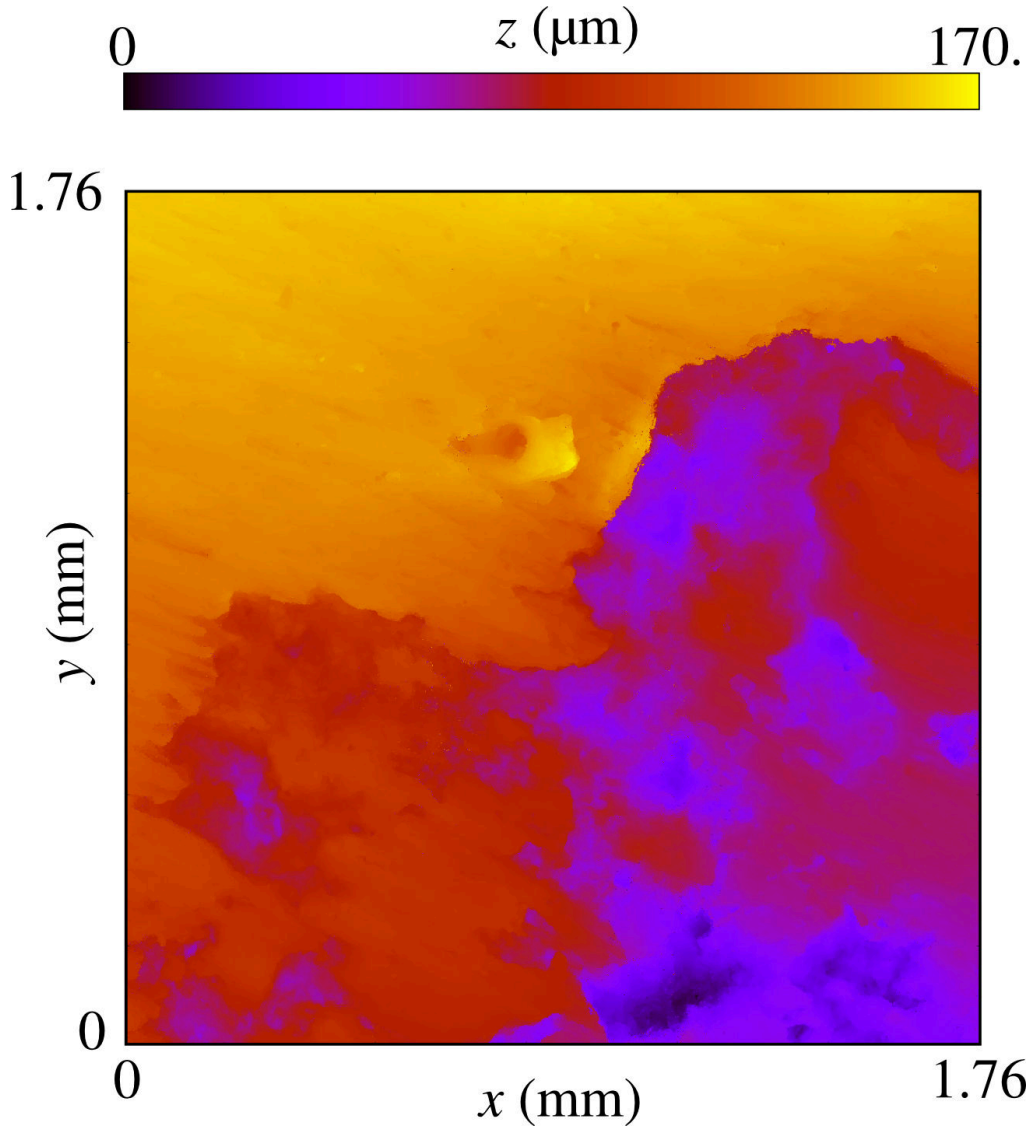
Multiscale road roughness

- Scale 2 (x5)



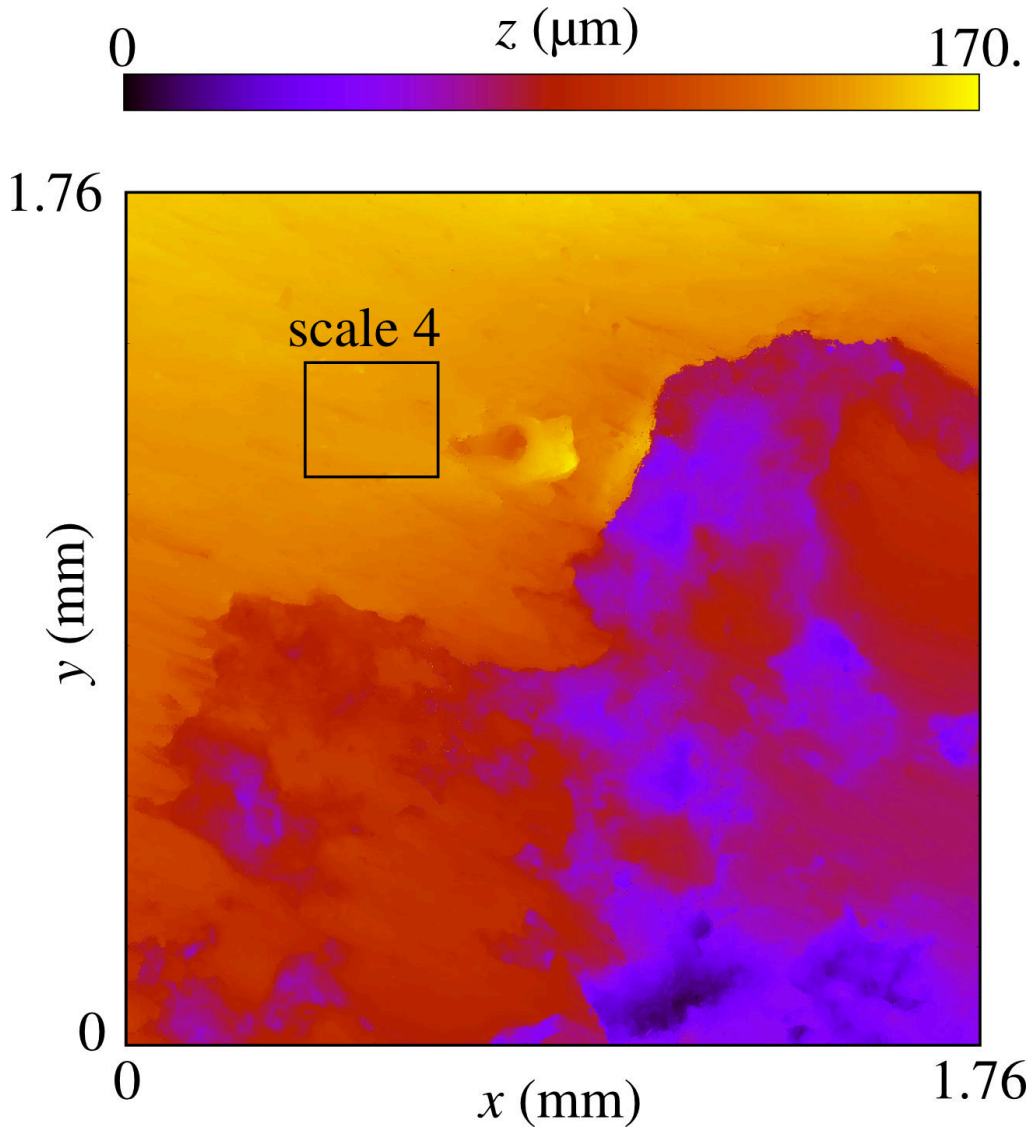
Multiscale road roughness

- Scale 3 (x20)



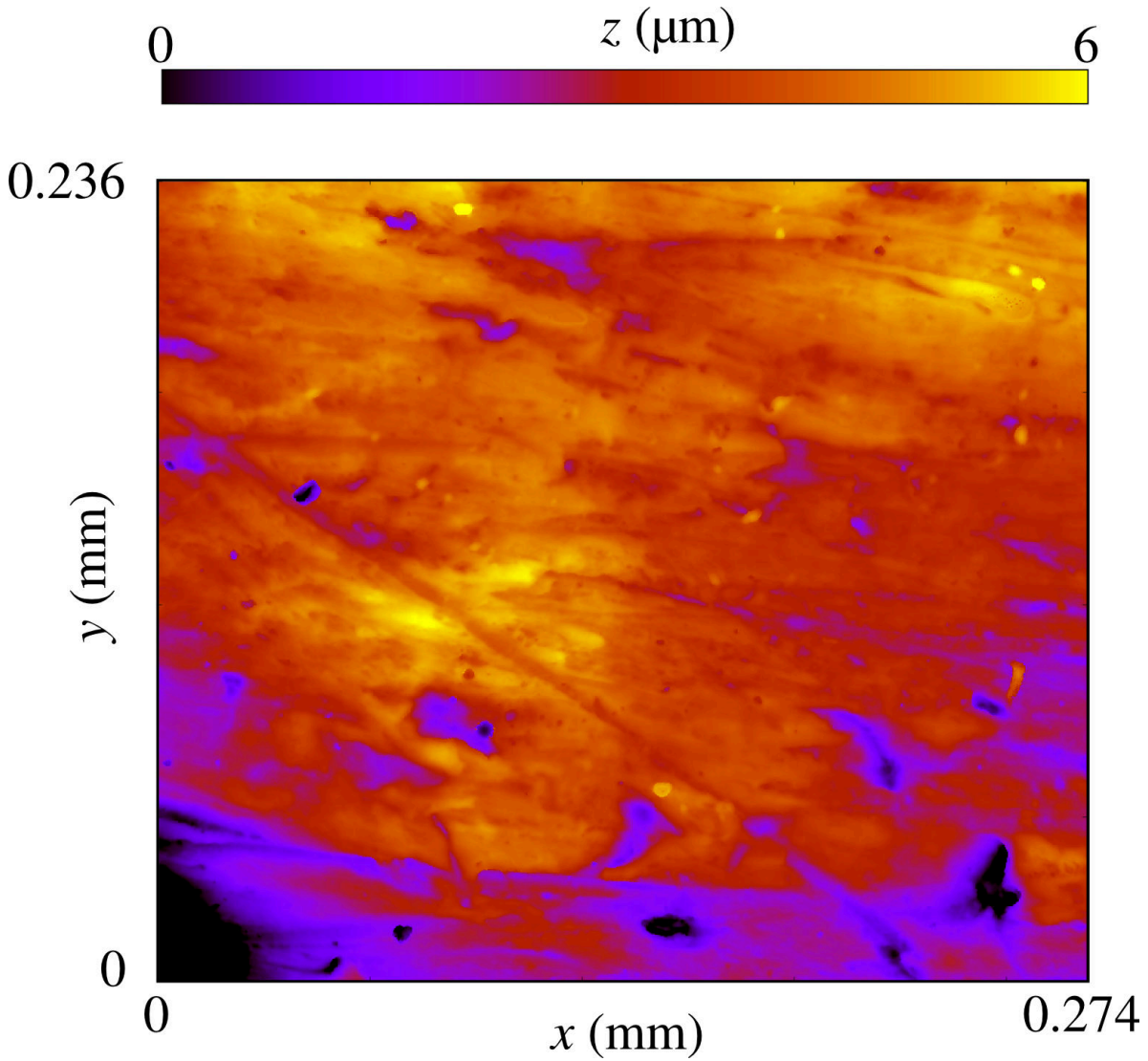
Multiscale road roughness

- Scale 3 (x20)



Multiscale road roughness

- Scale 4 (x100)



☺ Thank you for your attention!